MAGNETIC MONOPOLES WITH FRACTIONAL CHARGES ☆

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Magnetic monopoles may carry fractional charges either as a result of a non-zero vacuum angle or as a result of their coupling to fermion fields. We discuss the relationship between these two mechanisms in a SU(2) theory and in a generalization of the SU(5) grand unified theory. We give several arguments that support the claim that the monopole ground state in the SU(2) theory has the quantum numbers of half a fermion. We show that the minimal monopole in SU(5) may have fractional electric and $B-L$ charge even in a *CP* conserving theory but does not have a confined color electric charge. The confinement of magnetic monopoles in gauge theories with oblique confinement is briefly discussed.

Jackiw and Rebbi [1] have shown that the Dirac equation for an isospin-1/2 fermion in the presence of a soliton monopole has a single normalizable zero mode if the fermion acquires its mass through coupling to the isovector Higgs field which breaks the SU(2) gauge symmetry. Let us consider their model with the addition of a Dirac mass term for the fermion fields. The fermion couplings are then

$$\mathscr{L}_{\mathrm{f}} = \mathrm{i}\bar{\psi}_n(\not{D}\psi)_n + m_0\bar{\psi}_n\psi_n + h\bar{\psi}_n\boldsymbol{\phi}\cdot\boldsymbol{\tau}_{nm}\psi_m\,, \tag{1}$$

with

$$(\not{D}\psi)_n = \gamma^\mu(\delta_{nm}\partial_\mu - \tfrac{1}{2}\mathrm{i}e\tau^a_{nm}A^a_\mu)\psi_m\,.$$

The monopole solution is given by

$$A^0_a = 0\,,\quad A^i_a = \epsilon^{aij}\hat{r}_j A(r)/e\,,$$
$$\phi_a = \hat{r}_a\phi(r)/e\,. \tag{2}$$

At large r $\phi(r)$ approaches its vacuum expectation value, ν, and A goes to zero as $-1/r$. Both A and ϕ vanish at $r=0$.

The Dirac equation in the background fields (2) has one zero energy solution for $m \leqslant h\nu$ [2]. The implications of this zero mode for the quantum theory are obtained by second quantization of the Fermi fields in the fixed background (2). The expansion of the Fermi fields in terms of eigenfunctions of the Dirac equation leads to the conclusion that the ground state is two fold degenerate with

$$a|+\rangle = |-\rangle\,,\quad a^\dagger|-\rangle = |+\rangle \tag{3}$$

where $a^\dagger$, a are the creation and annihilation operators associated with the zero energy mode. Jackiw and Rebbi argue that the presence of a fermion number conjugation symmetry (when $m_0 = 0$ forces one to conclude that the states $|+\rangle$, $|-\rangle$ have fermion number $\pm 1/2$, since they differ in fermion number by one and should provide a representation of the fermion conjugation symmetry.

Given that the ground states carry fermion number $\pm 1/2$ one might also expect them to have the electric charge of half a fermion, i.e. $\pm e/4$ [3]. This possibility raises a number of questions. Since a non-zero vacuum angle θ induces an electric charge [4] we would like to understand the relation between the θ dependent charge and the charge deduced from the presence of the zero mode. Also, since the presence of a zero mode and the existence of a fermion number conjugation symmetry depend on the ratio of the Dirac mass to the Higgs mass we would like to understand how the spectrum changes as we vary this ratio. Finally, we would like to know how these phenomena generalize to more realistic theories such as the

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SU(5) grand unified theory of Georgi and Glashow.

A simple and powerful technique for obtaining qualitative information about the monopole ground state is Callan's [3] generalization of the bosonization method of Mandelstam [5] and Coleman [6]. The strategy is to keep only the $J = 0$ partial waves for the fermions and the dyon rotator as the relevant degrees of freedom. The problem then becomes $(1 + 1)$-dimensional and the fermions may be "bosonized" so that they appear as solitons in a $(1 + 1)$-dimensional generalization of the sine-Gordon model. Furthermore, the dyon rotator degree of freedom may be integrated out explicitly so that one is left with a lagrangian involving only the boson fields. It should be emphasized that the equivalence between the bosonized theory and the $J = 0$ fermion theory is fundamentally quantum mechanical. In particular a classical analysis of the bonon theory should not be expected to give detailed quantitative information about the fermion theory such as the radial dependence of charge distributions or branching ratios for fermion scattering. It should however be reliable for determining the ground state quantum numbers and symmetries since these are usually determined only by the requirement of finite energy and the symmetries of the fermion theory.

The boson lagrangian for the SU(2) theory with the fermion couplings (1) is in the notation of ref. [3]

$$\begin{aligned}\mathscr{L} = \int_0^\infty & \mathrm{d}r \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \tfrac{1}{2}\partial_\mu Q\partial^\mu Q \\ & - (e^2/32\pi^2 r^2)(\phi + Q - \theta/\sqrt{\pi})^2 \\ & + m_0\tfrac{1}{2}\mu(\cos 2\sqrt{\pi}\,\phi + \cos 2\sqrt{\pi}\,Q) \\ & + h\nu\tfrac{1}{2}\mu(\cos 2\sqrt{\pi}\,\phi - \cos 2\sqrt{\pi}\,Q)\end{aligned} \tag{4}$$

where we have included the effects of a $(\theta e^2/32\pi^2)\,F\tilde{F}$ term in the original lagrangian, μ is a normal ordering mass which will be unimportant in what follows. The boundary conditions that are equivalent to the boundary conditions in the original fermion theory (without the coupling ot the dyon degree of freedom) are

$$\phi(r=0) = Q(r=0)\,, \quad \phi'(r=0) = -Q'(r=0)\,. \tag{5}$$

As has been discussed recently [7,8] the dyon energetics require a modification of the second boundary condition to

$$\phi(r=0) + Q(r=0) = \theta/\sqrt{\pi}\,. \tag{6}$$

At $r = \infty$ ϕ and Q must minimize the mass terms in order to have a finite energy ground state. For $m_0 < h\nu$ this gives $\phi(r=\infty) = n\sqrt{\pi}$ and $Q(r=\infty) = (m+\frac{1}{2})\sqrt{\pi}$ with n, m integer. For θ sufficiently small the lowest energy configuration corresponds to $n = 0$ and $m = 0, -1$ and has electric charge and fermion number given by

$$Q = \left(\frac{e}{2\sqrt{\pi}}\right)[Q(r) + \phi(r)]\Big|_0^\infty = \pm e/4 - e\theta/2\pi\,,$$

$$F = (1/\sqrt{\pi})[Q(r) - \phi(r)]\Big|_0^\infty = \pm 1/2\,, \tag{7}$$

which recovers Jackiw and Rebbi's result of fermion number $\pm 1/2$ and also shows that the ground state has half the electric charge of a fermion for $\theta = 0$ [3]. For $m_0 > h\nu$ the boundary conditions at infinity are $\phi(r=\infty) = n\sqrt{\pi}$ and $Q(r=\infty) = m\sqrt{\pi}$ so that the states have charge and fermion number $(Q, F) = (\frac{1}{2}(n + m - \theta/\pi), n + m)$ corresponding to Witten's formula for the charge of the monopole ground state plus some number of fermionic excitations.

It may seem peculiar that for $m_0 < h\nu$ there are two degenerate ground states with charge $\pm e/4$ (at $\theta = 0$) while for $m_0/h\nu$ the ground state is unique and neutral. What happens to the charge $\pm e/4$ as we vary m_0? This situation is similar to the behavior of the θ dependent charge as the fermion mass goes to zero. If we work in a guage where $\langle\phi_a\rangle = \nu\delta_{a3}$ then the fermion mass matrix is

$$\begin{pmatrix} m_0 + h\nu & 0 \\ 0 & m_0 - h\nu \end{pmatrix}, \tag{8}$$

so that at $m_0 = h\nu$ there is a fermion with mass $2m_0$ and a massless fermion. The spatial variation of the field Q which carries the fractional charge is governed by $(m_0 - h\nu)^{-1}$ so as $m_0 \to h\nu$

the charge and fermion number at a fixed distance from the monopole core go to zero so that the fractionally charged states go over smoothly into the neutral ground state as m_0 is increased. This is consistent with the fact that the Dirac equation possesses a normalizable zero mode only for $m_0 < h\nu$.

The assignment of fermion number $\pm 1/2$ and charge $\pm e/4$ to the monopole ground state is also supported by the following considerations. The original fermion theory possesses a number of discrete symmetries in the fixed background fields (2). In the limit $m_0 \to 0$, $h \to 0$ the theory has a chiral symmetry with transformation

$$\psi \to \exp(i\alpha\gamma_5)\psi\,,$$
$$\phi \to \phi + a/\sqrt{\pi}\,, \quad Q \to Q + \alpha/\sqrt{\pi}\,. \tag{9}$$

Of course this symmetry is afflicted by an anomaly, $\partial_\mu J^\mu{}_5 \alpha e^2 F\tilde{F}$. This is correctly accounted for by the bosonized theory. We see that the transformation (9) does not leave the boson lagrangian invariant but changes the effective value of θ: $\theta \to \theta - 2\alpha$ as expected from the anomaly equation. For $m_0 = 0$ but $h \neq 0$ the theory has a fermion number conjugation symmetry

$$\psi_n \to \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix} \tau^2_{nm} \psi^*_m\,,$$
$$\phi \to Q + \sqrt{\pi}/2\,, \quad Q \to \phi + \sqrt{\pi}/2\,, \tag{10}$$

which changes the sign of the fermion number but leaves the electric charge unchanged. Note that this symmetry is *not* reflected in the ground state spectrum which has states with charge and fermion number $(Q, F) = (e/4, 1/2)$ and $(e/4, -1/2)$ at $m_0 = 0$ but does *not* have states with $(Q, F) = (-e/4, 1/2)$ or $(e/4, -1/2)$ as would be required by the above symmetry. The reason is simply that this fermion number conjugation symmetry is also anomalous since as is clear from (10) it involves a discrete chiral transformation and chiral transformations are anomalous. The fact that this symmetry is anomalous is also clear from instanton considerations. In this theory even when $m_0 = 0$ instantons will induce a Dirac mass term for the fermions which is not invariant under the transformation (10) [‡1]. For $h = 0$ but $m_0 \neq 0$ the theory has a fermion number conjugation symmetry which is not anomalous

$$\psi_n \to \begin{pmatrix} 0 & -\sigma_2 \\ \sigma_2 & 0 \end{pmatrix} \tau^2_{nm} \psi^*_m\,, \quad \phi \leftrightarrow Q\,, \tag{11}$$

and which is reflected in the spectrum of fermionic expectations of the ground state. Finally, with both h and m_0 non-zero but $\theta = 0$ the theory possesses a CP symmetry $\phi \to -\phi$, $Q \to -Q$ and it is this symmetry which forces the ground state to have half the quantum numbers of a fermion when the Dirac equation has a single zero mode.

The fractional electric charge can also be understood from the point of view of the "low energy" U(1) theory. If we have $m_0 < h\nu$ then this is a theory which has a fermion ψ_+ of charge $e/2$ and mass $m_0 + h\nu$ and a fermion ψ_- of charge $-e/2$ and mass $m_0 - h\nu$. If we start with $\theta = 0$ in the SU(2) theory then we will have $\theta_{\mathrm{QED}} = 0$ in the U(1) theory where we define θ_{QED} to be the coefficient of $(e/2)^2 \boldsymbol{E} \cdot \boldsymbol{B}/4\pi^2$ in the U(1) lagrangian. As usual we can make all fermion masses real and positive by performing a chiral transformation $\psi_- \to \exp(\pm\frac{1}{2}i\pi\gamma_5)\psi_-$. As a result of the anomaly, $\partial_\mu J^\mu{}_5 = e^2 \boldsymbol{E} \cdot \boldsymbol{B}/8\pi^2$ this changes θ_{QED} to $\pm\pi$. In this theory Witten's formula for the dyon charge is $Q_{\mathrm{D}} = e^2 g \theta_{\mathrm{QED}}/4\pi$. Since the fermions carry charge $e/2$ the minimal monopole charge satisfies $eg = 1$. In the theory with positive fermion masses and $\theta_{\mathrm{QED}} = \pi$ we therefore conclude again that the monopole ground state has electric charge $\pm e/4$ which is the minimum charge allowed by CP invariance.

The monopole ground state can have arbitrary fractional fermion number as well as electric charge if there are no discrete symmetries forbidding it. As an example consider the model of ref. [9] with fermion couplings

$$\mathscr{L}_{\mathrm{f}} = \bar{\psi}_n (m_0 \delta_{nm} + ih\boldsymbol{\phi} \cdot \boldsymbol{\tau}_{nm} \gamma_5) \psi_m\,. \tag{12}$$

The mass terms in the bosonized theory are

$$\mathscr{L}_{\mathrm{m}} = m_0 \tfrac{1}{2}\mu (\cos 2\sqrt{\pi}\,\phi + \cos 2\sqrt{\pi}\,Q) + h\nu\tfrac{1}{2}\mu (\sin 2\sqrt{\pi}\,\phi - \sin 2\sqrt{\pi}\,Q) \tag{13}$$

‡1 I thank E. Witten for pointing this out to me.

with $\nu = |\langle \boldsymbol{\phi} \rangle|$. To minimize the mass terms at $r \to \infty$ we require $\phi(\infty) = 1/2\sqrt{\pi}\,\tan^{-1} h\nu/m_0$ and $Q(\infty) = -1/2\sqrt{\pi}\,\tan^{-1} h\nu/m_0$ so that using (7) we find that the ground state has charge zero and fermion number $-1/\pi\,\tan^{-1} h\nu/m_0$ which agrees with the calculation of ref. [9]. In the U(1) theory the fermion masses can be made real and positive by the chiral transformation

$$\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix} \to \begin{pmatrix}\exp(\mathrm{i}\alpha\gamma_5) & 0\\ 0 & \exp(-\mathrm{i}\alpha\gamma_5)\end{pmatrix}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}, \tag{14}$$

with $\alpha = 1/2\tan^{-1} h\nu/m_0$. Since the corresponding current has no anomaly with the electromagnetic currents this transformation leaves θ_{QED} at zero and hence the monopole has charge zero.

As a final generalization of the SU(2) model we will consider what happens with more than one isodoublet of fermions. We will take N Dirac fermions which are isodoublets under SU(2) with Yukawa coupling

$$\mathscr{L}_{\mathrm{Y}} = h \sum_{\alpha=1}^{N} \bar{\psi}_{an} \boldsymbol{\phi} \cdot \boldsymbol{\tau}_{nm} \psi_{am}\,. \tag{15}$$

In addition to the SU(2) gauge symmetry and the overall U(1) of fermion number this theory has a SU(N) symmetry which could either be global or gauged. Since the monpole carries no SU(N) magnetic field there is no obstruction to defining SU(N) globally [10] and therefore the monopole states should form representations of SU(N). Again the bosonization trick makes the analysis particularly simple. We will now have $2N$ boson fields ϕ_a, Q_a, $a = 1, \ldots, N$ with lagrangian

$$\mathscr{L} = \int_0^\infty \mathrm{d}r \sum_a \left(\tfrac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a + \tfrac{1}{2}\partial_\mu Q_a \partial^\mu Q_a\right) + \frac{e^2}{32\pi^2 r^2}\left(\sum_a (\phi_a + Q_a)\right)^2 + \sum_a \frac{h\nu}{2}\,\mu\left(\cos 2\sqrt{\pi}\,\phi_a - \cos 2\sqrt{\pi}\,Q_a\right) \tag{16}$$

Let us first consider the limit $e \to 0$. The appropriate boundary conditions are then $\phi_a(0) = Q_a(0)$ and $\phi_a'(0) = -Q_a'(0)$. For $N = 1$ we have the two states $|+\rangle$, $|-\rangle$ shown in fig. 1 with fermion number $\pm 1/2$. For $N > 1$ the lowest

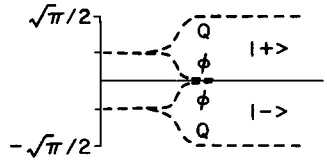

Fig. 1. Monopole ground states with fermion number $\pm 1/2$ for $e \to 0$.

lying states will correspond to each set of fields $\{\phi_a, Q_a\}$ being in one of the configurations $|+\rangle$ or $|-\rangle$ for a total of 2^N possible states. These decompose into states of definite fermion number as

$$2^N = \sum_{M=0}^{N} \binom{N}{M}, \tag{17}$$

where $\binom{N}{M} = N!/M!(N-M)!$ is the degeneracy of states with fermion number $(2M - N)/2$.

If we now let $e \neq 0$, the only change in the boundary conditions is that we now require $\Sigma_a\,[\phi_a(0) + Q_a(0)] = 0$ which just affects the behavior of the wave functions at the origin but leaves the behavior at infinity unchanged. The electric charge is just $Q = (e/2\sqrt{\pi})\,\Sigma_a\,[\phi_a(0) + Q_a(0)]|_0^\infty$ so that the $\binom{N}{M}$ states with fermion number $(2M - N)/2$ have electric charge $e(2M - N)/4$. Under SU(N) transformations these states transform as Mth rank antisymmetric tensors. The lowest energy states will be the states of lowest charge. For N even the smallest possible charge is zero so the ground state will have fermion number and charge zero but will transform as a $N/2$th rank antisymmetric tensor under SU(N). For N odd, $N = 2P + 1$, the ground state will have charge $\pm e/4$ and fermion number $\pm 1/2$ and will transform as a $(P+1)$th rank antisymmetric tensor or as its conjugate, a Pth rank antisymmetric tensor.

We will now consider to what degree the phenomenon discussed in the SU(2) model are relevant to monopoles in more realistic theories. For simplicity we will restrict ourselves to consideration of the minimal charge monopole in the SU(5) theory. We want to determine whether or not the monopole can carry fractional electric, color, or $B - L$ charge. Since the usual fermions acquire a mass through coupling to the **5** Higgs field which is topologically trivial in the monpole solution, we must enlarge the theory in order to have a fermion zero mode as in the SU(2) theory. The simplest possibility and the one we will consider is to add an additional **5** of *Dirac* fermions which acquire a mass by coupling to the **24** Higgs field:

$$\mathcal{L}_Y = h\bar{5}^i 24_i{}^j 5_j \,. \tag{18}$$

Since the mass of these fermions is of order the SU(5) breaking scale they have little effect on the standard SU(5) phenomenology.

We will write the **5** of Dirac fermions as $5_f = (D_1, D_2, D_3, E^+, N)$ in an obvious notation. The minimal charge SU(5) monopole lives in a SU(2) subgroup which mixes color and weak isospin [11] with the few fermions forming a doublet, (D_3, E^+), and three singlets. The coupling of the fermion doublet to the SU(2) triplet part of the **24** Higgs field is analogous to the Higgs coupling in the SU(2) model discussed previously so we expect one fermion zero mode in the monopole background field. This can also be shown to follow from an index theorem of Callias [2].

Again the charges are easy to extract in the bosonized theory. Outside the monopole core the adjoint Higgs field has the form

$$\mathrm{diag}(1,\, 1,\, -\tfrac{1}{4} + \tfrac{5}{4}\boldsymbol{r}\cdot\boldsymbol{\tau},\, -\tfrac{3}{2}) \,. \tag{19}$$

If we ignore all light fermions then the boson lagrangian just involves one boson field for each component of the doublet and is given by

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}\partial_\mu\phi_{\bar{D}_3}\partial^\mu\phi_{\bar{D}_3} + \tfrac{1}{2}\partial_\mu\phi_{E^+}\partial^\mu\phi_{E^+} \\ &\quad + (e^2/32\pi^2 r^2)(\phi_{\bar{D}_3} + \phi_{E^+} - \theta_5/\sqrt{\pi})^2 \\ &\quad + h\tfrac{1}{2}\mu(\nu\cos 2\sqrt{\pi}\,\phi_{\bar{D}_3} - \tfrac{3}{2}\nu\cos 2\sqrt{\pi}\,\phi_{E^+})\,,\end{aligned} \tag{20}$$

where θ_5 is the SU(5) θ parameter. If we take $\theta_5 = 0$ then the boundary conditions are

$$\begin{aligned}&\phi_{E^+}(0) = \phi_{\bar{D}_3}(0)\,,\\ &\phi_{\bar{D}_3}(\infty) = n\sqrt{\pi}\,,\quad \phi_{E^+}(\infty) = (m+\tfrac{1}{2})\sqrt{\pi}\} \quad h>0\,,\\ &\phi_{D_3}(\infty) = (n+\tfrac{1}{2})\sqrt{\pi}\,,\quad \phi_{E^+}(\infty) = m\sqrt{\pi} \quad h<0\,.\end{aligned} \tag{21}$$

If $h > 0$ the ground state configuration has the quantum numbers of half an electron or positron: color charge zero, electric charge $\pm e/2$ and $B - L = \pm 1/2$. If $h < 0$ it has the quantum numbers of half a d_3 quark or antiquark: color hypercharge $\pm 1/3$, electric charge $\pm e/6$ and $B - L = \mp 1/6$.

As in the SU(2) model we can understand the electric charges which correspond to magnetic fields carried by the monopole in terms of the θ parameters of the low energy theory. With unbroken $SU(3)_c \times U(1)_{em}$ there are two independent θ parameters, θ_{QCD} and θ_{QED}, which are defined as the coefficients of $(g^2/32\pi^2)F\tilde{F}$ in the QCD and QED lagrangians. When $h > 0$ we can make the heavy fermion masses real and positive by the chiral transformation $\psi_{E^+} \to \exp(\pm\tfrac{1}{2}\mathrm{i}\pi\gamma_5)\psi_{E^+}$. This leaves θ_{QCD} unchanged but shifts θ_{QED} by π. Witten's formula thus gives a dyon with electric charge $e\theta_{QED}/2\pi = \pm e/2$. If $h < 0$ the necessary chiral transformation is $\psi_D \to \exp(\pm\tfrac{1}{2}\mathrm{i}\pi\gamma_5)\psi_D$ which shifts θ_{QCD} by π and θ_{QED} by $\pm\pi\cdot 3\cdot(1/3)^2 = \pm\pi/3$. We therefore find a dyon with electric charge $\pm e/6$. Changing θ_5 and hence θ_{QCD} from 0 to 2π takes us from the monopole to the first dyon state which has the color hypercharge of a d_3 quark. Therefore $\theta_{QCD} = \pi$ gives a dyon with half the color hypercharge of a d_3 quark. Since this color electric field cannot be screened by quark or gluon fields it would appear that these monopoles must be confined by color forces. Unfortunately this monopole occurs in a world with $\theta_{QCD} = \pi$ and in the real world we know that θ_{QCD} must be close to 0, not π [12].

The effects of the light fermions and *CP* violation in the fermion mass matrices are easily included in this formalism. The charges of phenomenological interest are the color and electric charges. As we have seen the electric charges which correspond to magnetic fields carried by the monopole are exactly those expected from Witten's formula once all fermion masses are made real and positive by chiral transformations. Since θ_{QCD} is known to be very small in this basis the color electric field of the monopole will be very small and unconfined as is explained in ref. [10]. However θ_{QED} and therefore the monopole electric charge is completely unknown. If all CP violating phases are small the monopole electric charge should be close to either zero or $e/2$ corresponding to $\theta_{\mathrm{QED}} \simeq 0$ or π respectively. If strong CP violation is exorcised by the Peccei–Quinn mechanism then the color charge will be exactly zero but again the electric charge is undetermined since the instanton effects which force the axion field to cancel θ_{QCD} are independent of θ_{QED}.

In the penultimate paragraph it was suggested that the minimal SU(5) monopole may be confined at $\theta_{\mathrm{QCD}} = \pi$ by color forces. Although θ_{QCD} is small in the real world the θ parameters of other hypothetical gauge interactions such as technicolor may be non-zero so it is of some interest to determine whether or not monopoles will be confined in such theories. We will consider QCD at $\theta = \pi$. The arguments may be generalized to larger gauge groups. Very little is known about the behavior of confining gauge theories at non-zero θ, but following 't Hooft we can make a rough guess as to what kind of behavior we expect. 't Hooft [13] assumes that the QCD ground state can be described in terms of a set of transient dynamical variables which consist of one U(1) gauge field for each Cartan generator of SU(3), magnetic monopoles with respect to these U(1) fields, and electrically charged quarks and gluons. Following 't Hooft we take the various building blocks to have electric charges q_i and magnetic charges h_i, $i = 1, 2, 3$ with $\Sigma_i q_i = \Sigma_i h_i = 0$ to remove the overall U(1). The charges of the various states are given in table 1.

For $\theta \sim 0$ the monopoles M_1 and M_2 presumably develop vacuum expectation values which leads to a "dual" superconducting phase with confinement of the color electric fields of the quarks. For small θ the ratio of electric to magnetic charge for the condensed states (M_1, M_2) is the same as for the SU(5) monopole so we expect the color charge of the monopole to be screened. As we increase θ the condensed states must carry larger electric charges and it may become favorable to form neutral bound states so that the vacuum rearranges into an oblique confinement mode [13]. For $\theta = \pi$ the monopoles carry electric charge $\pm Q/2$ which is half the charge of a gluon so we may expect that $M_1M_1\bar{G}_1$ and $M_2M_2\bar{G}_2$ bound states develop vacuum expectation values. If this occurs then states that have charges that are linear combinations of those of the condensed states will be liberated while the other states will be confined. Since there are no electrically neutral combinations involving a single quark, quarks will still be confined. The SU(5) monopoles M will also be confined either in $M\bar{M}$ pairs or by binding to a quark $MM\psi_3$. The $M\bar{M}$ pairs will

Table 1
Oblique confinement in SU(3) at $\theta = \pi$. $Q = g/2$, $R = 4\pi/g$, $S = \theta g/4\pi$.

		h_1	h_2	h_3	q_1	q_2	q_3
gluons:	G_1	0	0	0	Q	$-Q$	0
	G_2	0	0	0	0	Q	$-Q$
monopoles:	M_1	R	$-R$	0	S	$-S$	0
	M_2	0	R	$-R$	0	S	$-S$
quarks:	ψ_1	0	0	0	$2Q/3$	$-Q/3$	$-Q/3$
	ψ_2	0	0	0	$-Q/3$	$2Q/3$	$-Q/3$
	ψ_3	0	0	0	$-Q/3$	$-Q/3$	$2Q/2$
condensed states:	$M_1^2\bar{G}_1$	$2R$	$-2R$	0	$2S-Q$	$Q-2S$	0
	$M_2^2\bar{G}_2$	0	$2R$	$-2R$	0	$2S-Q$	$Q-2S$
SU(5) monopole	M	$R/3$	$R/3$	$-2R/3$	$S/3$	$S/3$	$-2S/3$

eventually annihilate while the $MM\psi_3$ bound state should be stable and at large distances will look like a monopole with twice the Dirac unit of magnetic charge. It thus appears unlikely that a generalization of this mechanism could be used to solve the cosmological monopole problem since bound states of monopoles and fermions should form with roughly the same abundance as $M\bar{M}$ pairs.

To summarize we have given several arguments that show that the monopole ground state in the SU(2) theory is two-fold degenerate and carries the quantum numbers of half a fermion when the dominant contribution to the fermion mass is from coupling to the Higgs field. In general the electric charges of the monopole ground state which correspond to magnetic charges carried by the monopole are given by Witten's formula in terms of the θ parameters of the theory once all fermion masses are made real and positive. In the real world this implies that the minimal SU(5) monopole does not have a confined color electric field but may yield monopole confinement if the monopoles carry magnetic fields corresponding to other confining forces such as technicolor. The electric charge of this monopole is undetermined by present experiments and may be fractional ($\pm e/2$) even in a *CP* conserving theory. A measurement of this charge would yield important new information concerning the source of *CP* violation.

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