

# Effects of neutron-star superconductivity on magnetic monopoles and core field decay

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From the magnetic properties of old neutron stars we propose that an observation of a sufficiently old pulsar limits any "grand unified theory" heavy magnetic monopole flux in the pulsar neighborhood to below  $5 \times 10^{-24} \tau_{10}^{-2} \text{cm}^{-2} \text{sr}^{-1} \text{sec}^{-1}$ , where  $\tau_{10}$  is the age (in  $10^{10}$  yr) of the pulsar's present magnetic field and monopole speeds are  $\sim 10^{-3} c$ . For the millisecond pulsar PSR 1937 + 214 a major improvement over the Parker limit is obtained, which is also better than various limits from monopole catalysis of baryon decay, provided  $\tau_{10} \gtrsim 10^{-1}$ . The consideration of monopole dynamics inside superconducting neutron-star cores leads to this conclusion.

## I. INTRODUCTION

Many attempts to predict the form of some future grand unified theory of the elementary particles suggest that measurable fluxes of very massive magnetic monopoles should exist in the Universe. The typically predicted monopoles have masses ( $m$ ) of order  $10^{17} \text{GeV}/c^2 \sim 10^{-7} \text{g}$  and pole strengths ( $g$ ) equal to the minimal Dirac value,  $\hbar c/2e \sim 3 \times 10^{-8} \text{G cm}^2$ . In the neighborhood of a neutron star such monopoles are subject to both the enormous stellar magnetic field  $B$  (up to several  $\times 10^{12} \text{G}$  for young pulsars, and perhaps down to a few  $\times 10^8 \text{G}$  for what have been interpreted as  $10^9$ -yr-old neutron stars) and the huge stellar gravitational acceleration  $g_G \sim 10^{14} \text{cm sec}^{-2}$ . Since  $gB = 3 \times 10^4 B_{12} \ll mg_G \sim 10^7 m_{17}$  (where  $m_{17} \equiv$  monopole mass in units of  $10^{17} \text{GeV}/c^2$  and  $B_{12} \equiv$  the magnetic field in units of  $10^{12} \text{G}$ ), the neutron star's gravitational pull on the monopole is expected to dominate the surface magnetic force on it. Thus, a monopole can be pulled into the star and become permanently bound within it.

If the magnetic field were to penetrate into a normal stellar core, monopoles would accumulate near the stellar center, with the north and south monopoles separated by a distance  $(3gB_0/4\pi G\rho_0 m) \sim 10^3 B_{12} \rho_{15}^{-1} \text{cm}$ , where  $B_0$  is the central magnetic field and  $\rho_0$  the central density. However, because the protons in neutron stellar cores are expected to make a superconducting transition less than a century after the formation of the star no matter how hot the star was at birth, the magnetic properties of the core will be different from that of ordinary magnetized conducting matter in two essential ways.

(1) The superconducting core of the star is such a good diamagnet that a monopole cannot penetrate it unless either the monopole mass approaches the Planck mass ( $10^{19} \text{GeV}/c^2$ ) or there is a preexisting core magnetic field

which pulls the monopole into the core through normal (nonsuperconducting) regions.

(2) Where such a magnetic field exists in the superconducting core of neutron stars it is expected to be bunched into quantized flux tubes. Only by joining itself to flux-tube pairs can the monopole enter into the stellar core. Within such flux-tube pairs, the magnetic force hugely exceeds the gravitational one. As a result, magnetic monopoles gravitationally pulled beneath the surface of a neutron star have very different dynamics from that which holds when the superconducting stellar cores are absent. In particular, true equilibrium is never obtained for a  $10^{16}$ – $10^{17} \text{GeV}/c^2$  mass monopole within the stellar core. Given sufficient time, a few massive monopoles could, under circumstances discussed below, annihilate all of the flux which connects the core to the stellar surface. The energy to maintain monopole velocities despite energy dissipation to the ambient electron sea comes from the mobilization of the magnetic energy, released as the total flux-tube length diminishes, and from the energy of the normal magnetic flux-tube cores as they become superconducting.

In the present paper we analyze the resulting modified dynamics of monopoles inside neutron stars and the consequences on monopole capture. We also discuss briefly a new channel for core-field decay inside neutron stars.

## II. FLUX-TUBE STRUCTURE

In the stellar crust of a cooled neutron star, protons exist only within separated nuclei; in the outer crust only electrons fill the interstellar volume between nuclei, while in the inner crust both neutrons and electrons fill that volume. At a total density of about  $2 \times 10^{14} \text{g cm}^{-3}$  the very large neutron-rich nuclei have diameters  $\sim 10^{-11} \text{cm}$ .

The separation between nuclei is only a few fermis larger than the nuclear diameter, so the nuclei almost touch each other. There is a first-order phase change at about this density at which the nuclei "dissolve."<sup>1</sup> At higher densities, deeper within the star, protons are continuously distributed within a much denser neutron sea. As long as the temperature  $T \leq 10^9$  K ( $kT \leq 10^2$  keV), a temperature regime expected to be achieved within several years of the formation of a neutron star no matter how hot the star was initially, both the nuclear protons and the continuum core protons form BCS superconductors.

The possible existence of a charged-pion condensate in the center of the neutron stellar core depends upon the stellar mass and equation of state as well as on theoretical questions of some delicacy and controversy. If the central condensate does exist it forms a charged-boson superconductor when cooled below an expected superconducting transition temperature which is higher than that of the protons.

Magnetic flux which threads the superconducting matter of (a) the possible central charged-pion condensate, (b) the core protons and (c) the inner crust, will be organized in the following ways.

(a) In the pion condensate the magnetic field will be bunched into flux tubes, each carrying a flux

$$\Phi_\pi = \frac{2\pi\hbar c}{e} \sim 4 \times 10^{-7} \text{ G cm}^2. \quad (1)$$

This is exactly the total flux carried by the (minimum strength) Dirac monopole. The flux-tube radius is the London superconducting pion-condensate penetration depth,

$$\lambda_\pi \sim \left[ \frac{4\pi n_\pi e^2}{m_\pi c^2} \right]^{-1/2} \sim 10^{-12} \text{ cm}, \quad (2)$$

where  $n_\pi e^2/m_\pi$  is the ratio of the square of the pion-condensate charge density ( $en_\pi$ ) to its mass density ( $m_\pi n_\pi$ ). The flux-tube magnetic field is typically  $10^{16}$  G. The separation between flux tubes ( $d_\pi$ ) is determined by the average magnetic field in the condensate ( $\bar{B}$ ), unchanged by flux-tube bunching, and by  $\Phi_\pi$ :

$$d_\pi \sim \left[ \frac{\Phi_\pi}{\bar{B}} \right]^{1/2} \sim \frac{10^{-10} \text{ cm}}{(\bar{B}_{12})^{1/2}} \sim \frac{10^2 \lambda_\pi}{(\bar{B}_{12})^{1/2}}, \quad (3)$$

where the exact value for  $d_\pi$  depends on the "lattice structure" of the flux-tube arrangement. Because  $d_\pi \gg \lambda_\pi$ , lattice forces will be negligible and the flux tubes will be arranged as a two-dimensional fluid with an average distance given by the  $d_\pi$  in (3).

(b) In the core proton superconductor, the quantized flux ( $\Phi_p$ ) is half that carried in a pion condensate because the bosons are proton pairs; the flux is also half that of a putative Dirac monopole. A magnetic field which threads the superconductor will be bunched into such quantized tubes as long as the London penetration depth of the protons

$$\lambda_p = \left[ \frac{4\pi n_p e^2}{m_p c^2} \right]^{-1/2} \sim 10^{-11} \text{ cm} \quad (4)$$

( $n_p$  is the proton number density,  $m_p$  is the proton mass) exceeds  $2^{1/2}\xi_p$ , where  $\xi_p$  is the BCS correlation length of proton pairs (type-II superconductor). In the core,  $\xi_p$  is typically several  $10^{-13}$  cm, so that the flux-tube configuration of the magnetic field in the proton superconductor is expected to hold with a quantized flux of

$$\Phi_p = \frac{2\pi\hbar c}{2e} \sim 2 \times 10^{-7} \text{ G cm}^2. \quad (5)$$

The magnetic field structure of such a flux tube is

$$B \sim \frac{\Phi_p}{2\pi\lambda_p^2} \left[ \frac{\pi\lambda_p}{2r} \right]^{1/2} \exp \left[ -\frac{r}{\lambda_p} \right] \text{ for } r \gg \lambda_p, \quad (6a)$$

and

$$B \sim \frac{\Phi_p}{2\pi\lambda_p^2} \ln \frac{\lambda_p}{r} \text{ for } \xi_p < r \ll \lambda_p. \quad (6b)$$

At the average flux-tube separation in the core,  $d_p \sim 10^{-10}(\bar{B}_{12})^{-1/2}$  cm, the repulsive force between magnetic flux tubes, which falls off like  $\exp(-d_p/\lambda_p)$ , is negligible. The main force on a flux-tube segment in a region with constant  $\lambda_p$  is that from the huge flux-tube tension ( $\hat{T}_p$ ):

$$\begin{aligned} \hat{T}_p &\sim (\Phi_p^2/16\pi^2\lambda_p^2) \ln \left[ \frac{\lambda_p}{\xi_p} \right] \\ &\sim \frac{1}{2} \frac{B^2}{8\pi} \pi\lambda_p^2 \ln \left[ \frac{\lambda_p}{\xi_p} \right] \sim 10^8 \text{ dyn}. \end{aligned} \quad (7)$$

Because of proton density variations,  $\lambda_p$ , and thus  $\hat{T}_p$ , vary along a flux tube. But the surrounding exterior stellar fluid presses in on a tube whose surface normals are no longer perpendicular to its axis. The resulting axial force balances that from distant variations in  $\hat{T}_p$  so that only the local  $\hat{T}_p$  is relevant to the dynamics of flux-tube segments.

When describing the dynamics of monopoles, which in a stellar core superconductor must be a source or sink of flux tubes, it is convenient to consider the average flux-tube magnetic field

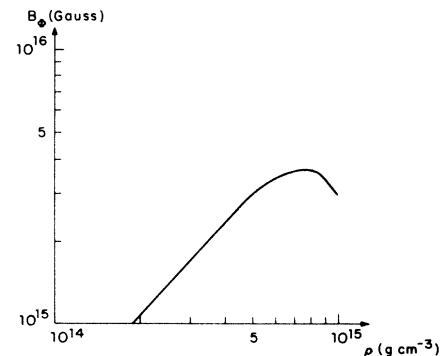


FIG. 1. Average magnetic field in a quantized flux tube in the proton-superconductor region of a neutron-star core [from proton density of R. Nandkumar and D. G. Ravenhall (private communication)].

$$B_{\Phi} \equiv \frac{\Phi_p}{\pi \lambda_p^2} = 2 \times 10^{15} \left[ \frac{n_p}{10^{37} \text{ cm}^{-3}} \right] \text{ G}. \quad (8)$$

This is plotted in Fig. 1 for estimated  $n_p$  between the transition layer to continuous protonic matter (at  $\rho \sim 2 \times 10^{14} \text{ g cm}^{-3}$ ) where  $B_{\Phi} \sim 10^{15} \text{ G}$ , and the stellar center, where it is around  $3 \times 10^{15} \text{ G}$ . Note that  $B_{\Phi}$  is a maximum, with  $B_{\Phi} \sim 3.6 \times 10^{15} \text{ G}$ , at  $\rho \sim 8 \times 10^{14} \text{ g cm}^{-3}$ , where the proton density is highest.

(c) Above the first-order transition surface, which separates the continuous proton fluid from protons clumped into discrete separated nuclei,  $\lambda_p \rightarrow \infty$  as long as proton-pair tunneling between nuclei is unimportant. Current detailed models of the transition indeed suggest that the large diamagnetic nuclei have radii ( $R$ ) comparable to, but somewhat smaller than, the penetration depth ( $\lambda_z$ ) of the superconducting proton fluid within the nuclei.<sup>2</sup> These nuclei have a diamagnetic permeability  $\mu$  given by

$$\mu - 1 = -8\pi \frac{\langle e^2 B^2 (x^2 + y^2) n_p \rangle}{8m_p c^2} \simeq -\frac{3R^2}{20\lambda_z^2}. \quad (9)$$

There will be some compression of a magnetic field threading the inner crust in the interstellar volume between nuclei, and some diminishing of it in the cores of nuclei. In first approximation, however, we shall consider the magnetic field above the transition layer to be near that which would exist there if the diamagnetic nuclei were absent [or, equivalently, that which would obtain if the inner crust were uniformly filled with a medium with susceptibility  $(\mu - 1)f$ , with  $f$  the nuclear filling factor].

The sudden (scale  $\lesssim \lambda_p$ ) transition between flux-tube confinement of  $B$  just below the nuclei-fluid transition to a  $B$  falling off nearly like the inverse square of the distance from the spout of the flux tube can occur only because of the surface energy of the transition layer ( $\sigma$ ), associated with a first-order transition. The departure of that surface from local planarity,  $z(x, y)$ , must be sufficient for that surface tension pressure  $\sigma \nabla_{x,y}^2 z$  to balance the part of  $\mathbf{B} \times (\nabla \times \mathbf{B})$  associated with the exit and entrance structure of a flux tube ("spout") which is not hy-

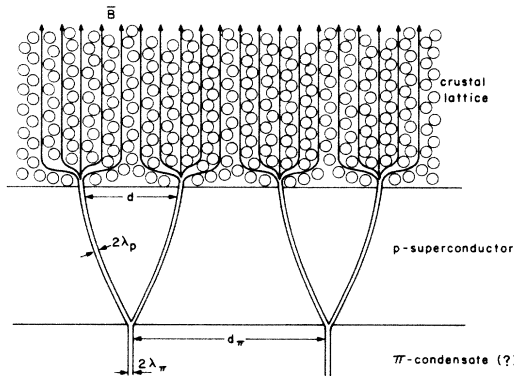


FIG. 2. Schematic representation of a neutron-star magnetic field penetrating through the lower crust, superconducting proton core, and a possible superconducting pion condensate. The flux-tube separation is  $d \sim 10^{-10}(\bar{B})^{-1/2} \text{ cm}$  and the proton superconductor depth extends for almost 10 km.

drostatically balanced. As long as  $\sigma \gg 10^{17} \text{ ergs cm}^{-2}$  or  $10^{-3} \text{ MeV}$  per surface nucleon,  $\sigma \gg \lambda_p B^2 / 8\pi$  at the transition surface. Such energy is quite negligible compared to total surface energy per nucleon; thus, in the spout of a flux tube, even just before reaching the transition layer, Eq. (8) is expected to remain a good approximation. Above the transition layer in the lower crust, at a distance  $r$  from the flux-tube spout, we have

$$B \sim \Phi_p / 2\pi r^2 \quad (10)$$

when the field modulation from the diamagnetic nuclei is ignored and  $r \ll d_p$  (the flux-tube separation). For  $r > d_p$ ,  $B \rightarrow \bar{B}$ , the average "classical" field. This spatial organization of a magnetic field which penetrates through the neutron star is idealized in Fig. 2.

### III. FLUX-TUBE MOTION

The neutron-stellar flux tubes are embedded in a highly viscous degenerate electron gas. They will generally move slowly through this gas for two different reasons.

(1) When flux tubes are curved, the huge tension of Eq. (7) will result in a concavely directed force per unit length

$$f_c = \frac{\hat{T}_p}{s_c}, \quad (11)$$

where  $s_c$  is the radius of curvature. This effort of a flux tube to achieve minimal length would move outward any flux tube whose exit and entrance spouts lie on a spherical surface and are movable.

(2) Because the hydrostatic pressure within a flux tube is less than that of the surrounding fluid by  $\delta_p \sim B_{\Phi}^2 / 8\pi$ , flux-tube matter is less compressed and will be locally buoyant. The buoyancy force per unit flux-tube length is

$$f_b \sim \frac{B_{\Phi}^2}{8\pi} \pi \lambda_p^2 \frac{g_G}{c_s^2}, \quad (12)$$

where  $c_s^2 = dp/d\rho$  is the squared sound speed and  $g_G$  the local gravitational acceleration. For  $s_c \sim R$  (the stellar radius) in Eq. (11), since  $R g_G \sim c_s^2$ , one finds that  $f_b \sim f_c$ . Note that if a flux tube is initially straight and parallel to the direction of the average magnetic field except for a very small "kink" close to the boundary to give "spouts" normal to the superconducting core surface, that geometry will change as it moves. At typical times in mid motion,  $s_c \sim R$  along the full vortex length, in part because of the variability of (12) and the electron drag along the vortex. Vortices closest to the magnetic-field axis will reach their maximum speed after the longest times.

If these flux-tube forces are not balanced by magnetic-field interactions, as seems to be the case for nonturbulent situations (see below) with  $d_p \gg \lambda_p$ , the velocity of the resulting flux-tube motion will be limited by the momentum transfer to electrons scattered off the magnetic field within a moving flux tube and by the Ohmic dissipation within the crust when the flux-tube spouts move. Crustal current relaxation has an uncertain time scale, estimated to be of order  $\lesssim 10^7 \text{ yr}$ . Spontaneous superconducting core flux-tube relaxation times, as discussed below, appear

to have a larger order of magnitude; hence, we shall consider only the latter as determining flux-tube relaxation rates.

The mean free path for electron scattering (mainly by phonons) in the proton-superconductor neutron-superfluid core of a neutron star is very much larger than a flux-tube radius. Hence, in their interaction with flux tubes, the electrons can be regarded as free. A flux tube of radius  $\lambda_p$  moving with velocity  $\mathbf{v}$  through the electron gas feels a force per unit length along  $\mathbf{v}$  of

$$\mathbf{f}_v \cong -4\pi^2 n_e p_e \mathbf{v} \lambda_p \langle 1 - \cos\theta \rangle, \quad (13)$$

where  $p_e \langle 1 - \cos\theta \rangle$  is the average momentum change along  $\mathbf{v}$  of the scattered electrons. There is also a (generally) much larger perpendicular force, proportional to  $\langle \sin\theta \rangle$ , which causes motion perpendicular to  $\mathbf{v}$  but does not affect the rate of flux-tube-length shortening or expulsion from the superconductor and hence is not important for our present discussion.

The neutron-star core electrons, which transverse a moving magnetic flux tube, have an inverse wave number

$$\lambda_e \cong \hbar/p_e \sim 3 \times 10^{-13} \text{ cm} \quad (14)$$

which is typically an order of magnitude less than the tube radius,  $\lambda_p$ . We shall, therefore, approximate the electron dynamics by classical trajectories. The ratio of the electron radius of curvature in the flux-tube magnetic field to the flux-tube radius

$$\begin{aligned} \frac{r_c}{\lambda_p} &\sim \frac{p_e c}{e B_\Phi \lambda_p} = \frac{\pi p_e c \lambda_p}{e \Phi_p} \\ &= 3 \left[ \frac{\lambda_p}{10^{-12} \text{ cm}} \right] \left[ \frac{E_f(e)}{80 \text{ MeV}} \right] \end{aligned} \quad (15)$$

[with  $E_f(e)$  the electron Fermi energy] is of order 10, so that the electron orbits are not bent greatly by their passage. Then

$$\langle 1 - \cos\theta \rangle \sim (\lambda_p / r_c)^2 \quad (16)$$

and the drag force per unit length is

$$|\mathbf{f}_v| \sim \frac{n_e e^2 \Phi_p^2}{E_f(e) \lambda_p} \left| \frac{\mathbf{v}}{c} \right|. \quad (17)$$

Magnetic flux-tube relaxation is driven by the forces of Eqs. (11) and (12). On balancing the drag of Eq. (17), one obtains a dissipative velocity along  $\mathbf{f}$  of

$$\frac{v}{c} \sim \left[ \frac{E_f}{4\pi n_e} \right] \left[ \frac{1}{2\pi \lambda_p R} \right] \sim 10^{-21} \quad (18)$$

and a flux-tube relaxation lifetime<sup>3</sup> of

$$\tau_R \sim \frac{R}{v} \sim 10^9 \text{ yr}. \quad (19)$$

The behavior of core magnetic fields on much longer time scales is not yet clear. Toroidal fields of flux tubes wrapped around poloidal ones or large multiple winding may result in the stabilization of at least parts of flux-tube systems within a core. Where flux tubes touch, their fields merge and the magnetic-field structure is much

closer to a classical one, where toroidal fields may stabilize poloidal ones.

We turn next to the behavior of magnetic monopoles which are either in the core when it becomes superconducting or gravitationally attracted into the star after that transition.

#### IV. MAGNETIC MONOPOLES IN SUPERCONDUCTING CORES

A Dirac monopole, already in the core of a neutron star when that core becomes a proton superconductor, must become the source or sink of a pair of quantized magnetic flux tubes. (In a possible central core pion condensate Dirac monopoles terminate single flux tubes.) The other terminal of the pair can be a monopole of opposite magnetic polarity. In that case the huge flux-tube tension of Eq. (7) will quickly (i.e., within seconds) pull the pole and antipole together, resulting in annihilation of the polarity. Alternatively, the flux-tube pair attached to a monopole in the core would reach the surface at very nearby points (the flux tubes, when formed, are only of order  $10^{-10}$  cm apart near the monopole if  $\bar{B} \sim 10^{12}$  G). The flux-tube pair would then pull the monopole on which they terminate to the surface where flux-tube quantization ceases and  $B$  drops over a distance  $d \sim 10^{-10} (\bar{B}_{12})^{-1/2}$  cm from the large  $B_\Phi \gtrsim 10^{15}$  G of the flux tube to the much smaller  $\bar{B}$ . As long as  $\bar{B}g < mg_G < B_\Phi g$ , which for  $\bar{B} \sim 10^{12}$  G corresponds to a monopole mass in the range  $10^{16} - 10^{18}$  GeV/ $c^2$ , the monopole will be ejected from the core but not escape from the crust; if  $\bar{B} < 10^{12}$  G, lower mass monopoles will also have the same fate. It is the subsequent motion of monopoles after they enter the crust that can involve detailed questions of some delicacy whose answers may depend upon a high velocity of monopole ejection out of a flux tube. (We shall see below that even monopoles can never be ejected fast enough to escape from the star.) For monopoles already in the core before the transition, the ejection velocity will be very different depending upon whether the central core region where the monopoles are originally trapped becomes superconducting before or after the region just below the core surface. In the former case, monopoles would be pushed up on the superconducting transition surface as it slowly (time scale of years) rises to its ultimate level at  $\rho \sim 2 \times 10^{14}$  g cm<sup>-3</sup>. The monopoles would, therefore, reach that ultimate surface with negligible velocity and could come to a static equilibrium supported by the surface tension of a shallow surface depression. It is then marginal whether a tangential component of  $B$  of order  $10^{12}$  G would then be able to move the monopole along that surface where it must pass through the alternating hills and depressions in the potential energy due to its interaction with the diamagnetic nuclei arranged in an ordered lattice above it. If, on the other hand, flux tubes first originate at  $\rho \sim 2 \times 10^{14}$  g cm<sup>-3</sup> and ultimately reach down to terminate on the monopole only as the superconducting transition penetrates down deeply enough, then the monopole will have the different subsequent behavior discussed in the next section. Unfortunately, the region of stellar density  $\rho \sim 2 \times 10^{14}$  g cm<sup>-3</sup> is just where calculation of the cooling time to the super-

conducting transition is rather delicate. While the transition temperature there increases with depth, the local neutrino cooling processes (time scales are generally too short for thermal conduction to be important) can also be less efficient with increasing depth. The most important of the standard ones are plasma neutrino emission, and, if the neutrons there are not yet superfluid, URCA emission. However,  $\rho \sim 2 \times 10^{14} \text{ g cm}^{-3}$  is the density near which the dominant neutron BCS pairing is changing from  $^1S_0$  to  $^3P_2$ . Thus, both neutron heat capacity and the URCA neutrino emission there depend sensitively on details and parameters that are poorly known.

We shall consider below the dynamics of a north magnetic monopole which is incident on the south polar region of a magnetized superconducting neutron-star core. The monopole may have been incident onto that side of the star initially or have been carried to it by the tangential component of the stellar magnetic field beyond the core. Because of the diamagnetism of the superconducting core, the moving monopole cannot penetrate the core until it is attracted to a south pole spout of an existing flux tube. Since the monopole carries twice the flux of a single flux tube it will also attract to itself the spout of a second flux tube (cf. Fig. 3). The flux-tube pair, whose average separation far from the monopole ( $d$ ) is  $2^{1/2}$  times that of Eq. (3), then pulls the monopole through the proton superconductor at a speed  $v$  determined by

$$df_v \sim 2\hat{T} \sim gB_\Phi \quad (20)$$

with  $f_v$  given by Eq. (17),  $\hat{T}$  by Eq. (7), and  $B_\Phi$  by Eq. (8). Then

$$\frac{v}{c} \sim \frac{E_f(e)}{n_e \lambda_p e^2 d} \sim 10^{-2} (\bar{B}_{12})^{1/2}. \quad (21)$$

Within a possible central superconducting pion condensate the monopole velocity exceeds that of Eq. (21) because the electron drag on a monopole terminating a single contracting flux tube is only that on the moving monopole. Its effective scattering cross section is  $\sigma \sim \hat{r}^2$ , where  $\hat{r}$  is that distance from the pole where the electron gyroradius in the magnetic field of the monopole equals  $\hat{r}$ . Then

$$\hat{r} = \frac{p_e c}{eB} = \frac{p_e c}{eg} \hat{r}^2 \quad (22)$$

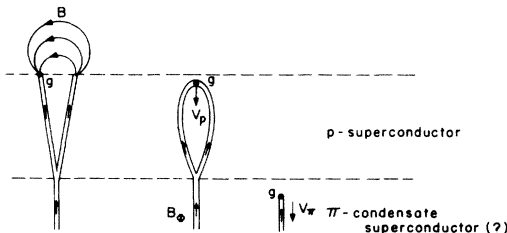


FIG. 3. Flux tubes capturing monopoles and pulling them through core superconductors.

and

$$\sigma \sim \left( \frac{eg}{p_e c} \right)^2 \sim \left( \frac{\hbar}{p_e} \right)^2. \quad (23)$$

The monopole-electron drag force  $F_v$  on a monopole of speed  $v$  is then

$$F_v \sim \left( \frac{\hbar}{p_e} \right)^2 n_e c p_e \left( \frac{v}{c} \right) \sim 10^{12} \left( \frac{v}{c} \right) \text{ GeV cm}^{-1} \quad (24)$$

within a stellar-pion-condensate core. The ratio of the monopole drag force in the proton superconductor to that in a pion condensate (i.e., that of a "free" monopole) is

$$\frac{df_v}{F_v} \sim \left( \frac{10^{-12} \text{ cm}}{\lambda_p} \right) \left( \frac{d}{10^{-13} \text{ cm}} \right) \sim 10^2. \quad (25)$$

However, an upper bound to the velocity of a monopole pulled by flux tubes is the Alfvén velocity ( $V_A$ ) within the flux tube. If  $v > V_A$  the monopole moves faster than the maximum speed at which the flux tube can shrink its length, and the flux-tube pull on the monopole is reversed: the monopole is dragged by the flux-tube magnetic wake that it leaves behind itself. In the core

$$V_A \sim \frac{B_\Phi}{(4\pi\rho)^{1/2}} \sim 10^7 - 10^8 \text{ cm sec}^{-1} \quad (26)$$

with the higher velocity limiting the monopole velocity deep within the core and the lower one limiting its exit velocity from the core. (Within a distance  $d \sim 10^{-10} \text{ cm}$  of the transition surface, the net flux-tube-pair force on the pole is greatly diminished because the two flux tubes no longer pull, even approximately, along a common direction and, indeed, pull in opposite directions just at the surface. However, the kinetic energy of the heavy monopole so greatly exceeds the energy lost to electron drag over that small distance that the monopole exit velocity is not significantly changed from its steady-state value at deeper levels.)

We can, therefore, conclude that (a) a monopole pulled through a stellar core will traverse the core in less than  $10^{-1} \text{ sec}$ , (b) in so doing it will catalyze the disappearance of a pair of flux tubes in the proton-superconducting region and a single flux tube in a possible central superconducting pion condensate, and (c) the exit velocity of the monopole is  $\lesssim V_A \sim 10^7 \text{ cm sec}^{-1}$  so that the monopole cannot achieve the escape velocity from the neutron star (even if kinetic-energy losses in transversing the crust are neglected).

We turn now to the critical question of the mobility of these monopoles in the presence of a magnetic field whose tangential component now pulls the monopole toward the opposite side of the neutron star where it can repeat the entire process—being pulled through the core and annihilating flux tubes.

## V. MONOPOLE MOTION ABOVE CRUST BOTTOM

Once the monopole exits the core and moves inside the crust, it is subject to the action of two types of forces mentioned above: (i) the electron drag, of order

$10^{11}(v_{\text{exit}}/c)$  GeV cm $^{-1}$ ; and (ii) the repulsion of the diamagnetic nuclei. Since the monopole enters the crust with kinetic energy  $\frac{1}{2}mc^2(v_{\text{exit}}/c)^2 \sim 10^{11}$  GeV, only after it traverses  $\sim 10^3$  cm will the electron drag forces produce a substantial effect. The free monopole will rise only to a height of  $(c^2/g_G)(v_{\text{exit}}/c)^2 \lesssim 1$  cm above the crust-core boundary before gravity will pull it down again. The energy loss to electron drag in the crust [ $\sim 10^{11}(v_{\text{exit}}/c)$  GeV cm $^{-1}$ ] in that cm is only  $\sim 10^{-3}$  the heavy monopole kinetic energy (assuming  $m \sim 10^{17}$  GeV/ $c^2$ ). Electron drag will become important only after many “bounces” up and down have taken place. (The first such bounce takes  $\sim 10^{-7}$  sec.)

The diamagnetic forces of the nuclei seem to be even less effective in damping the monopole motion. The monopole’s initial kinetic energy ( $\sim 10^7$  ergs) is enormously greater than its interaction energy with the lattice of diamagnetic nuclei through which it moves ( $\lesssim 10^{-4}$  ergs). The diamagnetic interaction energy of a monopole with a crust nucleus will not exceed  $(\mu-1)g^2/d_N \sim 10^{-1}$  GeV, where  $d_N$  is the nuclear separation, and the only monopole velocity regime where that interaction will be significant compared to the monopole kinetic energy is  $v \lesssim 10$  cm sec $^{-1}$ ; otherwise, the lattice will be adiabatically pushed slightly where necessary to “clear the way” as the monopole traverses it subsonically (shear velocity  $\sim 10^8$  cm sec $^{-1}$ ). The nucleus-nucleus Coulomb interaction energy,  $Z^2e^2/d_N$  is also only of order  $\sim 10^{-1}$  GeV, so that a slight adiabatic lattice deformation may indeed occur with no substantial reduction of the heavy monopole’s velocity.

The velocity of the monopole is not likely to decrease much below its exit velocity because, while bouncing up and down it is subject to the force of the tangential magnetic field in the crust. At  $v_{\text{exit}} \gtrsim 10^7$  cm sec $^{-1}$ , each bounce would last  $\lesssim 10^{-7}$  sec, and  $\tau_0 \gtrsim 10^{-4}$  sec will elapse before electron drag could be important. By then, however, the monopole will have acquired a tangential velocity in the tangential field,  $(B_t)_{12} \times 10^{12}$  G of

$$v_t \sim \frac{gB_t\tau_0}{m} \gtrsim 10^7(B_t)_{12} \text{ cm sec}^{-1}.$$

Even for  $(B_t)_{12} \sim 10^{-4}$ , this value of  $v_t$  is large enough for the monopole motion to be insensitive to the small periodic potential ripple from the lattice of nuclei, and electron drag will be the major source of energy loss. From equating the magnetic force with the electron drag force, the equilibrium velocity with which the monopole moves on the crust-core boundary surface becomes

$$v_t \sim 10^7(B_t)_{12} \text{ cm sec}^{-1}. \quad (27)$$

Where does the monopole go? That would naturally depend very much on the exact magnetic-field configuration on the boundary surface. For a smooth field configuration, the monopole will move all the way to the other side of the star, where it is able to attach itself to new flux-tube spouts and start the whole process again. It will take the monopole a time

$$\tau \sim (B_t)_{12}^{-1} \text{ sec} = 10^4(B_t)_8^{-1} \text{ sec} \quad (28)$$

[where  $(B_t)_8 = 10^4(B_t)_{12}$ ] to do that, which will then be

the single monopole recycling time needed for the annihilation of one flux-tube pair by one monopole. However, Eq. (28) may be modified by the nonsmooth arrangement of the tangential field at the crust-core surface. Such nonsmoothness will arise from two important factors.

(i) As the field decays [by relaxation processes such as these leading to Eq. (19)] a concentration of field lines around the equator of the crust-core boundary surface is likely to develop leading to some turbulent field configurations. The monopole may traverse this region very fast, as  $B_t$  will be quite large there; but even if it gets trapped there it will be able to destroy pairs of flux tubes at typical times  $\ll \tau$  per pair.

(ii) The core field may be extremely turbulent or complex to start with, so that the total length and/or the total number of (tangled) vortex lines could far exceed those implied by the average field. However, relevant reconnection times are such that turbulence should not have much effect on time scale  $\gtrsim 10^8$  yr.

We thus find that neutron stars may lose their magnetic fields if embedded in a sufficiently large external magnetic monopole flux. Some neutron stars exist, however, which are expected to be quite old, but still possess substantial magnetic fields. In the next section we discuss the upper limit posed on galactic monopole fluxes from the existence of these neutron stars.

## VI. LIMITS ON MONOPOLE FLUX FROM PULSAR OBSERVATIONS

Limits on the galactic monopole flux have been obtained on the assumption that monopoles catalyze neutron decay:



(with a similar stimulated decay for neutrons). Monopoles, trapped in the extremely high density inside neutron stars, might induce sufficient nucleon decay for the thermal energy released to be detectable even for a relatively small number of trapped monopoles. Freese, Turner, and Schramm<sup>4</sup> have calculated a total neutron-star luminosity of

$$L \simeq 1.3 \times 10^{35} F_{-16} \tau_6 R_{15}^{-2} M_{1.4}^2 (\sigma\nu)_{-28} \times \beta_{-3}^{-2} \text{ ergs sec}^{-1} \quad (30)$$

from

$$N_m \simeq 8 \times 10^{16} \tau_6 R_{15} M_{1.4} \beta_{-3}^{-2} F_{-16} \quad (31)$$

monopoles, where  $F_{-16}$  is the monopole flux in units of  $10^{-16}$  cm $^{-2}$  sr $^{-1}$  sec $^{-1}$  (the Parker limit),  $\tau_6$  is the monopole collection time in units of  $10^6$  yr,  $R_{15}$  and  $M_{1.4}$  are the neutron-stellar radius and mass in units of 15 km and  $1.4M_\odot$ , respectively, the cross-section factor  $(\sigma\nu)_{-28}$  is  $(\sigma\nu) \times 10^{28}$  cm $^{-2} \times (3 \times 10^{10})^{-1}$  cm $^{-1}$  sec,  $\beta_{-3}$  is the monopole galactic velocity in units of  $3 \times 10^7$  cm sec $^{-1}$  and  $f$  is a geometrical factor of order unity. Altogether, one monopole generates  $\sim 10^{18}$  ergs sec $^{-1}$  inside a neutron star.

Limits on  $F_{-16}$  utilizing Eq. (30) can come either from the diffuse x-ray background<sup>5</sup> or from selected individual

neutron stars. Based on the Helfand, Ruderman, and Shaham<sup>6</sup> data on pulsar 1929 + 10, Freese, Turner, and Schramm<sup>4</sup> put a limit

$$F_{-16} \leq 7 \times 10^{-6} \left[ \frac{\tau_6}{3} \right]^{-1} \left[ \frac{L}{2.6 \times 10^{30} \text{ ergs sec}^{-1}} \right] \times (\sigma\nu)_{-28}^{-1} R_{15}^4 \beta_{-3}^2 M_{1.4}^{-2} f^{-1} \quad (32)$$

on the monopole flux. This bound may come down by a factor of  $\sim 10^6$  from monopoles trapped during the pre-neutron-star phases of 1929 + 10 and possible  $M\text{-}\bar{M}$  annihilations at the neutron-star formation stage.

One notes that (32) is some 5 orders of magnitude below the Parker limit (and some 11 orders below the Cabrera one). However, it does depend on the ability of monopoles to induce nucleon decay. In the present paper we present a limit depending only on the monopole's mass. From the discussion in the previous sections, a neutron star which is  $\tau_{10} \times 10^{10}$  yr old and which still possesses a surface field of  $B_8 \times 10^8$  G (with no evidence for any past field regeneration processes) cannot contain more than

$$N_m \lesssim 3 \times 10^9 B_8 R_{15}^2 \tau_{10}^{-1} \tau \quad (33)$$

monopoles. This limits the average monopole flux during  $\tau_{10}$  to less than

$$F_{-16} \lesssim 5 \times 10^{-12} B_8 R_{15} \tau_{10}^{-2} M_{1.4}^{-1} \beta_{-3}^2 \tau, \quad (34)$$

where, in (33) and (34),  $\tau$  is to be taken from Eq. (28). On making this substitution and identifying  $B_i \sim B$  (in order of magnitude) we have

$$F_{-16} \leq 5 \times 10^{-8} R_{15} \tau_{10}^{-2} M_{1.4}^{-1} \beta_{-3}^2, \quad (35)$$

a rough semiquantitative limit which is, however, altogether independent of  $B$ . (This is, of course, no surprise: the larger  $B$  is, the faster the single monopole recycling time, but also the larger the number of vortex lines that need to be annihilated.)

Since (35) depends rather strongly on  $\tau_{10}$  and not at all (or, very slightly) on  $B_8$ , the natural candidates for obtaining a minimum monopole flux limit will be the oldest pulsars, as long as their fields are sufficiently large ( $B_8 \gtrsim 10^{-2}$ ) so as not to allow monopole interstitial trapping by the diamagnetic nuclei in the lattice. Indeed, even neutron stars which are not pulsars may qualify if it can be shown that they have  $B_8 \gtrsim 10^{-2}$ . However, it seems that pulsars will always satisfy this condition. Theory and observations both agree that pulsar radiation mechanisms operate only when  $B_8/P^2 \gtrsim 10^4$ , where  $P$  is the pulsar rotation period, in seconds. On the other hand, to prevent centrifugal instability of the pulsar, one must have  $P \gtrsim 10^{-3}$  sec. Hence  $B_8 \gtrsim 10^{-2}$  is a general condition for pulsing, and we can conclude that (35) can be applied to any old pulsar. For pulsars less than  $10^7$  yr old the electrical conductivity of the crust would maintain an initial external magnetic field even if the entire core field were to be annihilated.

At present, it seems that ordinary pulsars are no more than  $\sim 10^7$  yr old (and have fields  $B_8 \gtrsim 10^4$ ) so that (35) does not give a limit which improves on the Parker limit. These limits are not as good as those coming from monopole catalysis of nucleon decay; however, they do not depend on the existence of catalysis, only on the magnetic charge (and mass regimes of the monopole.)<sup>7</sup>

Much better limits can be obtained by considering a new class of pulsars, the so-called "millisecond" pulsars (see, e.g., Ruderman and Shaham<sup>8</sup>). These appear to be reprocessed neutron stars, one of which (1937 + 214) is indicated to be  $\gtrsim 10^9$  yr old, with  $B_8 \gtrsim 1$ . Here we obtain, from (35),

$$F_{-16} \lesssim 10^{-7} R_{15} M_{1.4}^{-1} \beta_{-3}^2, \quad (36)$$

which is an even better limit than (32) and independent of the catalysis mechanism. This is the main result of the present paper. Pre-neutron-star phase considerations will, again, push this limit down by several additional orders of magnitude, but here one needs to know in detail the evolution of the system, which is a problem still under investigation.

Needless to say, (36) depends critically on the  $\tau_{10}$  value one ascribes to 1937 + 214. While its spin-down age does seem to be  $\sim 2 \times 10^8$  yr and its overall age may be much longer,<sup>8</sup> its kinetic-age (i.e., its distance above the galactic plane divided by its velocity) measurement has now only a lower limit of about  $10^5$  yr.<sup>9</sup> It is not clear what that measurement can tell us about the evolution of 1937 + 214, but until this point is settled one should also keep in mind, that the pulsar could, perhaps, albeit with low probability, simply have been born with a small field and we just happen to view it when it is extremely young. In this case one may want to apply (35) to 1953 + 29, which may be several  $10^7$  yr old so that a limit  $\leq 10^{-3}$  times that of (36) for the neutron-star phase.

We note that if PSR's 1937 + 214 and 1953 + 29 are less than  $10^6$  yr old, then their present dipole magnetic fields might still have a contribution from crustal currents generated during their accretion spin-up phases.<sup>10</sup> In this case even a zero core field would not be incompatible with observations so that no monopole bound would follow.

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<sup>1</sup>For the less neutron-dominated nuclei of the collapsing stellar core, which is usually assumed to be the immediate predecessor of a neutron star, the nuclei pass through a great variety of shapes and sizes as the transition is approached [J. Lattimer (private communication); D. G. Ravenhall, C. J. Pethick, and J. R. Wilson, *Phys. Rev. Lett.* **50**, 2606 (1983)].

<sup>2</sup>J. Lattimer (private communication).

<sup>3</sup>This time is much longer than the estimate of A. G. Muslimov and A. I. Tsygan [Soviet Academy Report No. 877, 1984 (unpublished)], for the buoyancy forces of Eq. (12). The difference lies mainly in an additional factor of  $kT/E_f$  in their estimate for the electron drag opposing the tube motion [our Eq. (17)]. Such a factor would give a nonphysical temperature-proportional resistivity for a degenerate electron sea moving through randomly placed scatterers. Note that the estimate (19) assumes independent single-vortex motions. This will not be the case when flux-tube number density increases so as to support collective motions; one should then consider coherent electron scattering, as their mean free path will grow beyond the intervortex spacing. The classical (very long time) limit will prevail.

<sup>4</sup>K. Freese, M. S. Turner, and D. N. Schramm, *Phys. Rev. Lett.*

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<sup>5</sup>E. W. Kolb, S. A. Colgate, and J. A. Harvey, *Phys. Rev. Lett.* **49**, 1373 (1982); S. Dimopoulos, J. Preskill, and F. Wilczek, *Phys. Lett.* **119B**, 320 (1982).

<sup>6</sup>D. J. Helfand, M. A. Ruderman, and J. Shaham, *Nature (London)* **304**, 423 (1983).

<sup>7</sup>One might question if and how the possible existence of catalyzed nucleon decay might modify this result. Its major effect seems to be the heating up of a microscopic spherical region around the moving monopole, resulting in the quenching of superconductivity there, but giving no important change in any of the above time scales. While the steady-state temperature profile around the monopole depends on the local heat conductivity and neutrino cooling, the total heat generated ( $\sim 10^8 N_m \text{ erg sec}^{-1}$ ) will leave the average stellar core temperature well below that for the superconducting transition.

<sup>8</sup>M. A. Ruderman and J. Shaham, *Comm. Astrophys.* **10**, 15 (1983).

<sup>9</sup>M. M. Davis, J. H. Taylor, J. M. Weiseberg, and D. C. Backer, *Nature (London)* **315**, 547 (1985).

<sup>10</sup>R. D. Blandford, J. H. Applegate, and L. Hernquist, *Mon. Not. R. Astron. Soc.* **204**, 1025 (1983).