CP VIOLATION AND MASS RELATIONS IN SO_{10} ☆

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We note that in SO_{10} a large complex Majorana mass for the right-handed neutrinos provides high temperature soft *CP* violation. We then present a natural three-family SO_{10} model which reproduces observed mass relations and mixing angles and lacks strong *CP* violation in lowest order. The *CP* violation of the $K_0-\bar{K}_0$ system is unrelated to the high temperature *CP* violation mentioned above.

The simplest grand unification of color and electroweak forces is certainly achieved by the SU_5 model [1]. Attractive [2] as it may be, it has several distasteful aspects all related to the reducibility of the fermion representation: some particles are in different representations than their antiparticles, some fermions do not appear in the fundamental representation and anomalies are miraculously cancelled between representations. These aesthetic objections can be removed in a model based on the anomaly-free group SO_{10} [3]. There, each family of left-handed fermions belongs to a complex 16-dimensional spinor representation. Assuming the t-quark exists, the known fermions can be described in terms of three such families. Under $SO_{10} \supset SU_5 \times U_1$, each family decomposes as $\underset{\sim}{16} = \underset{\sim}{\bar{5}} + \underset{\sim}{10} + \underset{\sim}{1}$, where one recognizes the familiar "Woolworths" assignment of SU_5 plus a neutral singlet. This singlet plays the role of a right-handed neutrino and represents the greatest obstacle to the credibility of SO_{10}. It has been suggested [4,5] that this singlet be given a large Majorana mass, of the order of the unification mass, by means of a Higgs transforming according to the complex $\underset{\sim}{126}$ representation of SO_{10}. Then the mechanism that gives the charge 2/3 quarks their masses mixes these extra neutral states with the usual left-handed neutrinos, giving (per family) a neutrino mass matrix of the form

$$\begin{pmatrix} 0 & a \\ a & A \end{pmatrix},$$

where a is of the size of the $SU_2 \times U_1$ breaking and A is of the size of the breakdown of SO_{10} into SU_5. This results in a small mass of order a^2/A for the left-handed neutrinos. The appearance of neutrino masses is to be expected in view of the irreducibility (per family) of the fermion representation [5]. In SU_5 the fermion reducibility allowed for global conservation of $B - L$, thus forbidding neutrino masses.

We note that this mechanism has the potential of explaining the creation of excess baryon number in the early universe. Consider an embryonic SO_{10} model with one left-handed 16_f of fermions, one 45_H, and one 126_H of Higgs. The only Higgs coupling to fermions is given by $16_f \cdot 16_f \cdot \overline{126}_H$. We assume that all couplings are real so that the theory is *CP* invariant before spontaneous breaking (neglecting SO_{10} instanton effects). Both 45_H and 126_H must acquire vacuum expectation values (vev) to break SO_{10} down to $SU_3^c \times SU_2 \times U_1$. We take the 126_H vev to be an SU_5 singlet; its phase is determined by the presence in the potential of a $(126_H)^4 + (\overline{126}_H)^4$ term. With a coupling constant of the proper sign this gives a vev with a 45° phase. This effect produces *CP* violation in the model. The decay of the heavy 126_H into fermion pairs in general violates *CP* and baryon number. In this way one has all the necessary ingredients to produce baryon asymmetry in a manner that does not depend on the

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$SU_2 \times U_1$ breaking [6]. This is particularly satisfying since the scales at which electroweak breaking and baryon number production occur are so different.

We now present a realistic three-family SO_{10} model which embodies this mechanism. It also incorporates naturally the approximate mass relations [7] (at the unification scale) $m_\tau = m_b$, $m_\mu = 3m_s$ and $m_e = \frac{1}{3}m_d$, and the relation $\tan^2\theta_c \approx m_d/m_s$, where θ_c is the Cabibbo angle. Also the b-quark is predicted to decay preferentially to charm.

The model consists of three families of left-handed fermions, $16_1, 16_2, 16_3$, corresponding to the e, μ and τ families, respectively, two real 10's of Higgs, 10_1 and 10_2, three 126's of Higgs, $126_1, 126_2, 126_3$, and one 45_H of Higgs. The model is not asymptotically free but it is temporarily free between 1 GeV and the unification scale. The Yukawa couplings are given by

$$\begin{aligned}&(A\, 16_1 \cdot 16_2 + B\, 16_3 \cdot 16_3)\overline{126}_1\\ &\quad + (a\, 16_1 \cdot 16_2 + b\, 16_3 \cdot 16_3)(10_1 + \mathrm{i}\, 10_2)\\ &\quad + c\, 16_2 \cdot 16_2\, \overline{126}_2 + d\, 16_2 \cdot 16_3\, \overline{126}_3 + \text{h.c.},\end{aligned}$$

where A, B, a, b, c, d are real coupling constants. Naturalness is enforced by two global U_1's which will be broken spontaneously. Goldstone bosons are avoided by quartic terms in the potential of the form $(126_1)^4 + (\overline{126}_1)^4$, $(10_1 + \mathrm{i}\,10_2)^4 + (10_1 - \mathrm{i}\,10_2)^4$, $(126_2)^4 + (\overline{126}_2)^4$. These break the two global U_1's leaving discrete symmetries sufficient to preserve the naturalness of the Yukawa couplings. The SU_5 decomposition of the $\underset{\sim}{126}$ is given by

$$\underset{\sim}{126} = \underset{\sim}{1} + \underset{\sim}{\overline{5}} + \underset{\sim}{10} + \underset{\sim}{\overline{15}} + \underset{\sim}{45} + \underset{\sim}{50}\,.$$

We assume there is a range of parameters in the potential such that (1) the 126_1 acquires a large SU_5-singlet complex vev which gives all right-handed neutrino Majorana masses and breaks *CP* at unification scales, (2) the 126_2 acquires a vev along the $\underset{\sim}{45}$ of SU_5, (3) the 126_3 acquires a vev along the $\underset{\sim}{\overline{5}}$ of SU_5, (4) both 10_1 and 10_2 acquire vev's. The strengths of the 126_2, 126_3, 10_1 and 10_2 breaking are of the order of 300 GeV. The resulting mass matrices in the e, μ, τ family basis are

$$\begin{pmatrix} 0 & R\mathrm{e}^{\mathrm{i}\theta} & 0 \\ R\mathrm{e}^{\mathrm{i}\theta} & S\mathrm{e}^{\mathrm{i}\chi} & 0 \\ 0 & 0 & T\mathrm{e}^{\mathrm{i}\theta} \end{pmatrix} \quad \text{for charge } -1/3 \text{ (quarks)}\,,$$

$$\begin{pmatrix} 0 & P\mathrm{e}^{\mathrm{i}\delta} & 0 \\ P\mathrm{e}^{\mathrm{i}\delta} & 0 & Q\mathrm{e}^{\mathrm{i}\mu} \\ 0 & Q\mathrm{e}^{\mathrm{i}\mu} & U\mathrm{e}^{\mathrm{i}\delta} \end{pmatrix} \quad \text{for charge } 2/3 \text{ (quarks)}\,,$$

$$\begin{pmatrix} 0 & R\mathrm{e}^{\mathrm{i}\theta} & 0 \\ R\mathrm{e}^{\mathrm{i}\theta} & -3S\mathrm{e}^{\mathrm{i}\chi} & 0 \\ 0 & 0 & T\mathrm{e}^{\mathrm{i}\theta} \end{pmatrix} \quad \text{for charge } -1 \text{ (leptons)}\,.$$

These yield the mass relations and mixing angles stated above. The argument of the determinant of the quark mass matrix depends only on the 10_1 and 10_2 vev's. By choosing the sign of terms such as the $(10_1 + \mathrm{i}\,10_2)^4$ in the potential, the vev's of 10_1 and 10_2 can be orthogonal, thus yielding a real determinant of the quark mass matrix. This explains [8] the lack of strong *CP* violation provided that the QCD vacuum is (strong) *CP* invariant.

We note that no quadratic or cubic terms are needed in the Higgs potential for global symmetry breaking and/or naturalness reasons. This allows for a purely quartic potential à la Coleman–Weinberg [9]. The presence of two scales necessary to break SO_{10} down to $SU_3^c \times SU_2 \times U_1$ raises interesting possibilities. One is to break (with the 126_1) SO_{10} down to SU_5 and then at a lower scale break (with the 45_H) SU_5 down to $SU_3^c \times SU_2 \times U_1$. This route reproduces the SU_5 values for the Weinberg angle [12] and proton decay rate [2,10]. Another more intriguing route is to break SO_{10} to $SU_4 \times SU_2 \times U_1$ by means of the 45_H and then to $SU_3^c \times SU_2 \times U_1$ by means of the 126_1. This has the virtue of yielding a slightly higher [11][‡1] Weinberg angle without significantly affecting proton decay. At the same time it gives a higher mass to the left-handed neutrinos. We expect that detailed calculations of baryon number generation will constrain the relative scale of the two breakings [12]. Finally we note that the model gives *CP* violation in the left-handed charged current [13], neutrino oscillations as well as very small $\mu \to e\gamma$ and neutrinoless double β-decay rates. Detailed analysis and predictions of this model will be published elsewhere [14].

‡1 These authors also saw the need for the 126 in order to obtain a satisfactory mass relation, but failed to present a model with very light neutrinos and a sufficiently unstable b-quark, and *CP* violation.

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