Cosmological Baryon-Number Generation in Grand Unified Models

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(Received 4 March 1981)

Methods for complete calculation of cosmological baryon-number generation in the hot big-bang early universe are outlined and are applied to several SU(5) models. Effects of several baryon-number—nonconserving bosons and the presence of nonthermalizing modes are treated.

PACS numbers: 98.80.-k, 12.20.Hx, 95.30.Cq

Cosmology is potentially an important source of information on the baryon-number-density–nonconserving interactions expected in most grand unified gauge models. Any net baryon number density (B) imposed as an initial condition on the universe should have been rapidly destroyed by any B-nonconserving interactions. To account for the observed ratio of baryon number density to photon number density, $n_B/n_\gamma \simeq 10^{-9}$, a net baryon number must subsequently have been generated. This requires, in addition to B nonconservation, the violation of C and CP (and hence T) invariance, along with departures from thermal equilibrium.[1,2] This Letter outlines the complete calculation of n_B/n_γ generation in specific grand unified models in the context of the standard hot big-bang model of the early universe. The method we present allows for the exact treatment of an arbitrary number of superheavy bosons as well as the presence of nonthermalizing modes.[3] We summarize results for several realistic SU(5) models. Many details and extensions are discussed by Harvey *et al.*[4] in another paper.[4]

We denote heavy bosons generically by χ and light fermions by $a, b, \ldots$. The number density n_i of a particle i, and that of its antiparticle $n_{\bar{i}}$ are parametrized by $i_+ \equiv (n_i + n_{\bar{i}})/n_\gamma$ and $i_- \equiv (n_i - n_{\bar{i}})/n_\gamma$. The time development of these quantities is described by a set of coupled Boltzmann transport equations. For the heavy bosons these are[2,4]

$$\dot{\chi}_+ = -\sum_{a,b} \langle \Gamma(\chi \to ab) \rangle (\chi_+ - \chi_+^{\mathrm{eq}}), \tag{1a}$$

$$\dot{\chi}_- = -\sum_{a,b} \langle \Gamma(\chi \to ab) \rangle [\chi_- - (a_- + b_-)\chi_+^{\mathrm{eq}}], \tag{1b}$$

where dots denote time derivatives and the expansion of the universe is accounted for through division by n_γ in the definitions of $i_\pm$. The first terms on the right-hand side of Eqs. (1a) and (1b) correspond to free decays of χ and $\bar{\chi}$ with partial rates $\langle \Gamma(\chi \to ab) \rangle$ averaged over the decaying χ energy spectrum. The second terms account for back reactions in which the χ decay products interact to produce χ. The equilibrium number density χ_+^{eq} is obtained by integrating the $\exp(-E_\chi/T)$ equilibrium Maxwell-Boltzmann phase-space density. In equilibrium, $\chi_+ = \chi_+^{\mathrm{eq}}$ and $\dot{\chi}_+ = 0$; the expansion of the universe produces deviations from equilibrium at temperatures $T \sim m_\chi$.

The densities of fermion species develop according to

$$\dot{f}_- = \sum_{a,b,\chi} \langle \Gamma(\chi \to ab) \rangle (N_f)_{ab} [(\chi_+ - \chi_+^{\mathrm{eq}}) R(\chi \to ab) + 2\chi_- - (a_- + b_-)\chi_+^{\mathrm{eq}}] + \sum_{a,b,c,d,\chi} n_a [(N_f)_{ab} - (N_f)_{cd}](a_- + b_- - c_- - d_-) \langle |v| \sigma_\chi'(ab \to cd) \rangle, \tag{2}$$

where $(N_f)_{ab}$ denotes the number of particles of type f in the state ab. $R(\chi \to ab)$ denotes the difference in branching ratios between the CP conjugate decays $\chi \to ab$ and $\bar{\chi} \to \bar{a}\bar{b}$ divided by the full rate for χ decay; it vanishes if CP is conserved. The first part of the first term on the right-hand side of Eq. (2) therefore represents the production of an asymmetry in fermion number density as a result of CP-nonconserving decays of a symmetrical χ, $\bar{\chi}$ mixture. The second part causes asymmetries, χ_-, between χ and $\bar{\chi}$ to be transferred to the fermions when the $\chi(\bar{\chi})$ decays. The third part gives a correction to the rate for inverse decays resulting from the deviation of the fermion number densities from their equilibrium value. The second term in Eq. (2) represents the production and destruction of fermions by two-to-two scattering processes. σ_χ' is the cross section for this scattering mediated by χ exchange, but with the term corresponding to a real intermediate χ removed (since this is already accounted for by χ decay and inverse decay processes).

The number of independent particle densities to be treated in Eqs. (1) and (2) may be reduced by using unbroken symmetries (gauge and global). For non-Abelian groups, any asymmetries are shared symmetrically among members of each irreducible representation. If only a subset of the interactions that may potentially contribute to Eq. (2) are included there may be additional symmetries leading to further conserved combinations of fermion number densities (e.g., Π conservation in the absence of Higgs-fermion couplings for the models discussed below).

Let $f_-^{\,i}$ $(i=1,\ldots,N_f)$ be the independent fermion asymmetries and $\chi_-^{\,\alpha}$ $(\alpha=1,\ldots,N_\chi)$ the independent supermassive boson asymmetries. It is convenient to form a set $\vec{Q}$ which consists of independent quantum number densities B, L, etc. related to $\vec{F}=\{f_-^{\,i},\chi_-^{\,\alpha}\}$ by a unitary transformation, $\vec{Q}=H\vec{F}$, $\vec{F}=H^{-1}\vec{Q}$.

The thermalization of a quantum number Q_i through reactions of a particular boson χ is given from Eq. (2) by $\dot{Q}_i=\sum_\chi \chi_+^{\,\mathrm{eq}} M_{ij}^{\,\chi} Q_j$, where

$$M_{ij}^{\,\chi}=\sum_{k,l}\Delta Q_i(\chi\to f^k f^l)\langle\Gamma(\chi\to f^k f^l)\rangle(H_{kj}^{\,-1}+H_{ij}^{\,-1})$$

and $\Delta Q_i(\chi\to f^k f^l)$ represents the change in the value of Q_i through the reaction $\chi\to f^k f^l$. Boltzmann's H theorem requires that the eigenvalues of M^χ are all real and nonpositive. Any zero eigenvalues reveal additional symmetries; the corresponding eigenvector of number densities is then conserved in χ reactions [e.g., Π in vector boson exchanges in SU(5)].

We consider two grand unified models based on SU(5). In each case a family of fermions transforms as a reducible representation $(\underline{5}^*\oplus\underline{10})_i$, labeled by the family index i. The following Higgs representations are taken to couple to fermions: in model I [minimal SU(5)], a single $\underline{5}$ of Higgs, H_5; in model II, H_5 and an additional $\underline{5}$ of Higgs, $H_{5'}$. The Yukawa couplings in these models have the schematic form $[\underline{5}_i^*(D_\alpha)_{ij}]H_\alpha+[\underline{10}_i(U_\alpha)_{ij}\underline{10}_j]\times\bar{H}_\alpha$.

It may be shown that a CP-nonconserving nonzero $R(\chi\to ab)$ enters through an imaginary part of the product of the couplings in diagrams in which one boson is exchanged between the ab produced in the χ decay. The sum over a and b in Eq. (2) runs over all types and families of fermions; thus for fixed fermion types $R(\chi\to ab)$ is proportional to a family-space trace of Yukawa coupling matrices. In model I the first diagram exhibiting CP nonconservation involves only Higgs bosons and is of eighth order in the Yukawa couplings.[4-6] It is proportional to the imaginary part of the family-space trace, $\mathrm{Tr}[UU^\dagger UD^2U^\dagger D^2]$, suggesting the rough estimate $R\sim10^6h^8\epsilon$, where $|\epsilon|\leq1$ and h is an averaged Yukawa coupling at unification scales. The naive expectation that h will increase at low energy scales may be invalid if $h\geq g$, since the renormalization-group equation for h will have positive and negative contributions of roughly equal size.

In model II, both H_5 and $H_{5'}$ have only the single B-nonserving component,[7] $(3,1,\frac{1}{3})$; since $\underline{5}$ is a complex representation one may form complex linear combinations so that the $(3,1,\frac{1}{3})$ in both $\underline{5}$ and $\underline{5}'$ is separately a mass eigenstate. This suffices to show that no CP nonconservation may occur for gauge boson decay with Higgs scalar exchange or vice versa. CP nonconservation may occur at $O(h^2)$ through $\underline{5}$ decay with $\underline{5}'$ exchange (and vice versa).[8]

$\mathrm{SU}(3)\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$ symmetry allows the fifteen independent fermion fields in a family of an SU(5) model to be reduced to the set U_L, $(U^C)_L$, $(D^C)_L$, E_L, and $(E^C)_L$ (the subscript L denotes the left-handed helicity state and C denotes charge conjugation). The model contains a $(3,2,\frac{5}{6})$ of B-nonconserving vector bosons X (with number densities parametrized by X_- and X_+). We consider the case where there are n_S (=1 or 2) scalars, $S_1,S_2,\ldots,S_{n_S}$, transforming as $(3,1,\frac{1}{3})$ (with number densities parametrized by S_{i-} and S_{i+}). These models possess a locally conserved weak hypercharge whose initial value we assume to be zero. The models exhibit two further zero eigenmodes. The first is $B-L$ which has zero eigenvalue (is conserved) in all boson interactions. A second zero eigenmode, $\Pi=-3(D^C)_{L-}-2E_{L-}$, is present if scalar-fermion interactions are removed.[3] Π (termed "fiveness") corresponds to the net number density of the fermion species appearing in the $\underline{5}$ representation.

A density Π_0 generated through Higgs decays would be distributed as $B = -\Pi_0/10$, $\nu_- = -\Pi_0/5$ through Π-conserving X interactions. Π_0 may be destroyed through exchanges of light Higgs bosons. A convenient choice of independent combinations of fermion densities is $n_B/n_\gamma \equiv B = 2D_{L-} - (U^C)_{L-} - (D^C)_{L-}$, Π, and $\nu_- = E_{L-}$.

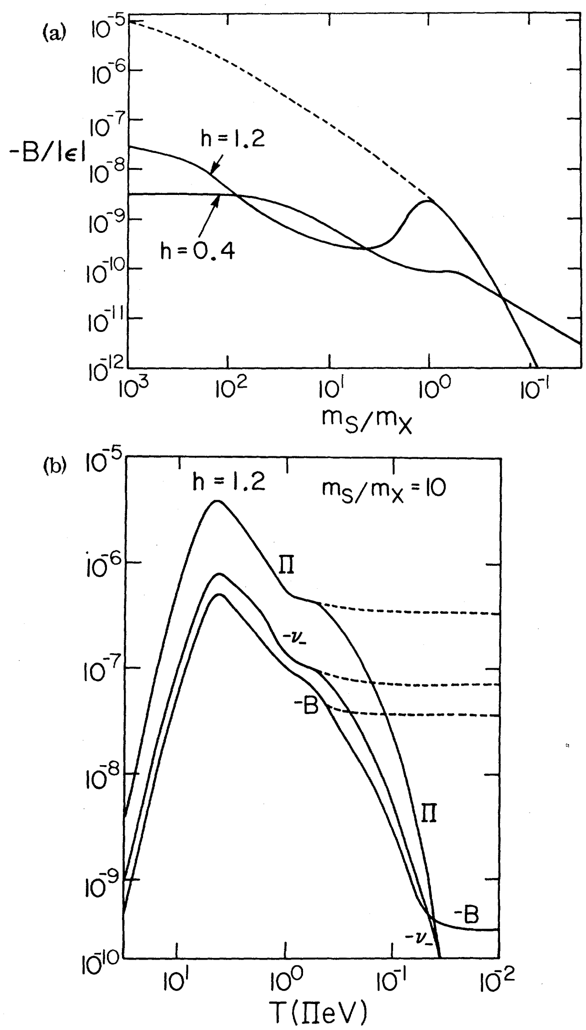

FIG. 1. (a) Baryon number density as a function of the Higgs boson (S) mass generated in the minimal SU(5) model in which the Yukawa coupling is h. Results are for $\alpha = \frac{1}{40}$, $m_X = 5\times10^{14}$ GeV. The CP nonconservation parameter ϵ is unknown but less than 1. (b) Evolution of independent quantum number densities as a function of temperature in the minimal SU(5) model. B denotes the net baryon number; ν_-, the asymmetry between $\bar{\nu}$ and ν densities; and Π, the total asymmetry between fermions in the $\underline{5}$ and $\underline{5}^*$ representations of SU(5); 1 TTeV $= 10^{24}$ eV. In these graphs the parameter ϵ has been scaled out. The dashed curves are results obtained by neglecting light Higgs boson exchange processes.

For model I, according to the estimate for $R(S \to ab)$ given above, an adequate baryon-number asymmetry will be generated only if $h = O(1)$, as would be the case if very heavy fermions exist with masses $\sim m_W$. Similar conclusions have been reached by Segre and Turner.[9] Figure 1(a) shows the baryon asymmetry (taking $m_X = 5\times10^{14}$ GeV and $\alpha = \frac{1}{40}$) as a function of m_S/m_X for $h = 1.2$ and $h = 0.4$ obtained by numerically integrating the Boltzmann transport equations (1) and (2). When $m_S/m_X \gg 1$, X exchanges thermalize the B produced in S decay to the value $-\Pi/10$; meanwhile, Π is reduced by light Higgs boson interactions. The final B attained is determined by the reduction in Π that occurs before X exchanges cease to be important and B becomes fixed. For $m_S/m_X < 1$ the X is not effective in destroying the baryon number built up through S decay. The enhancement in the final value of B around $m_S/m_X = 1$ is a result of the transition between these two regions. The dashed curve shows the final baryon number if all X interactions are artificially set to zero. Figure 1(b) shows the temperature development of the quantum number asymmetries B, Π, and ν_- for the case $h = 1.2$, $m_S/m_X = 10$, with the solid (dashed) curves indicating the effect of including (excluding) the destruction of Π and ν_- by the interactions of the light Higgs doublet. The final B is reduced from the value of R given above due to the light Higgs boson exchanges.

For model II the final baryon number density as a function of m_{S_1}/m_X is shown in Fig. 2 for differ-

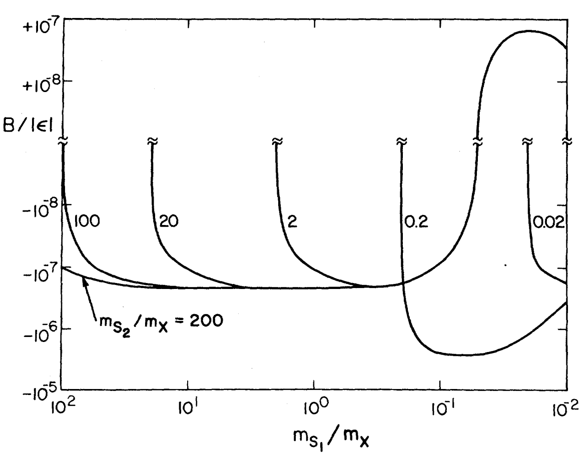

FIG. 2. Baryon number density for an SU(5) model with two baryon-number–nonconserving Higgs bosons (S_1, S_2) as a function of the S_1 mass for different choices of the S_2 mass. The results are for $\alpha = \frac{1}{40}$ and $m_X = 5\times 10^{14}$ GeV. The CP nonconservation parameter ϵ is unknown but less than 1.

ent choices of m_{S_2}/m_X. Note that, when $m_1 = m_2$, we have [assuming $(\Gamma_{S_1})_{\rm total} = (\Gamma_{S_2})_{\rm total}$ in the Born approximation] $R(S_1 \to ab) = -R(S_2 \to ab)$ and hence no B is generated. For $m_{S_i} > m_X$ the additional decay mode $S_i \to X + \varphi$ (where φ is a light Higgs boson) decreases the effective CP nonconservation, $R(S_i \to ab)$, in S_i decay. For $m_{S_2} > m_X$ and $m_{S_1} > m_X$, the final B is negative and determined by vector thermalization of the positive Π produced in S_2 decay. For $m_{S_2} > m_X$ but $m_{S_1} < 0.1 m_X$, the final baryon number is positive and determined mainly by inverse decays into S_1. The dominant term governing the time evolution of B for $T \gtrsim m_{S_1}$ is $\dot{B} \propto S_{1+}{}^{\rm eq}\langle \Gamma_{S_1}\rangle(14\nu_- - 12B + 7\Pi)$ with similar equations for $\dot{\nu}_-$ and $\dot{\Pi}_-$. Since $\Pi > 0$, $\Pi > \nu_-$, and $\Pi > B$, this term tends to drive B positive. In general there are three linear combinations of B, ν_-, and Π which decrease as exponentials until cut off at temperatures below m_{S_1}. The final value of B thus depends sensitively on the initial values of Π, ν_-, and B. For this reason, it is inadequate to assume that B is produced and damped in successive independent stages as in simple models which treat only one quantum number.[2,10] For both $m_{S_2} < m_X$ and $m_{S_1} < m_X$ inverse decays into S_1 are no longer able to change the sign of the negative B produced through S_2 decays and hence the final B produced is negative. The possibility of changes in the sign of B associated with detailed features of the boson spectrum indicates that no generic relation may be found between the definition of "matter" as given for the K^0-$\bar{K}^0$ system and that determined from the cosmological baryon-number asymmetry.

One of the authors (E.W.K.) wishes to thank G. Segre and M. Turner for conversations at the Aspen Center for Physics. We would also like to thank T. Goldman for discussions.

This work was supported in part by the U. S. Department of Energy under Contract No. DE-AC-03-79ER0068, by the Fleischman Foundation, and by the National Science Foundation under Grant No. PHY79-23638.

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DISCOVERY OF A 6^-, $T=1$ RESONANCE IN ^{24}Mg VIA HIGH-RESOLUTION INELASTIC ELECTRON SCATTERING. H. Zarek, B. O. Pich, T. E. Drake, D. J. Rowe, W. B. Bertozzi, S. Kowalski, F. N. Rad, P. Sargent, C. F. W. Williamson, and R. A. Lindgren [Phys. Rev. Lett. 38, 750 (1977)].

The excitation energy of the 6^-, $T=1$ resonance in ^{24}Mg was 15.130 ± 0.04 MeV and not 15.045 ± 0.04 MeV.