OPEN STRINGS ON ORBIFOLDS

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We consider propagation of type I SO(32) superstrings on orbifolds. It is shown that anomaly cancellation requires the existence of "twisted" open strings and we determine the Chan–Paton factors for these strings in a simple example.

1. Introduction. String theories can be divided into two classes, open and closed. It is clear that the closed string theories, especially the class of heterotic string theories [1], are the more appealing, both from a phenomenological and a philosophical viewpoint. However, there has been much progress in the study of open string theories, and at present the type I SO(32) superstring appears to be a fully consistent theory [2].

In order to make contact with low-energy physics it is necesssary to understand the propagation of strings in background fields. So far this has been studied mainly from the first-quantized point of view and in the context of closed strings. It would be useful to be able to study these questions in string field theory. Since at present open string field theory is better understood than closed string field theory it would be useful to have explicit examples of non-trivial solutions to the open string equations of motion.

The simplest compactifications of the heterotic string which have phenomenological possibilities are the class of orbifold compactifications [3]. The background consists of a flat torus T with points identified under the action of a discrete space group S. The total symmetry group G of relevance consists of the combined action of S and an embedding of S, $\tilde{S}$, in $E_8 \otimes E_8$ or SO(32). Conformal invariance is immediate since the string degrees of freedom are just free two-dimensional fields with boundary conditions determined by the structure of G. The physical states of the theory are those states that are invariant under G. Hence we construct the Hilbert space for closed strings on T and then project onto states invariant under G. However, it is also necessary to consider strings on T that are closed only up to identification of points by a nontrivial group element, i.e. $X^I(\sigma+\pi) = gX^I(\sigma)$. The fermion fields, both spacetime and internal, will satisfy similar boundary conditions. The states in these twisted Hilbert spaces must also be projected onto G invariant states. The above arguments have a very natural interpretation from the point of view of the path integral. The spectrum can be found by considering the one-loop vacuum functional. If the background is an orbifold, then the path integral should be performed over world sheets with the topology of the torus with the world sheet fields identified by some element of G around each noncontractible loop. Projecting onto states invariant under G is equivalent to summing over tori with different boundary conditions in the τ direction. However, in order for the theory to be modular invariant, it is necessary to consider tori twisted in the σ direction as well, since there is a global reparameterization of the world sheet that exchanges τ and σ. Hence modular invariance imposes powerful constraints on the theory.

This is all well and good for closed strings. However, the story is different for a theory of open and closed strings. One-loop open string

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world-sheets are annuli and Möbius strips and we no longer have modular invariance as a guiding principle. Furthermore, the closed strings that one considers are non-orientable, which means that it is necessary to include the Klein bottle in the one-loop path integral in order to project onto states invariant under $\sigma \to \pi - \sigma$. The analog of the modular group for the Klein bottle can be viewed as the automorphism group of the fundamental group of the Klein bottle. The fundamental group of the Klein bottle is non-abelian with two generators, a and b, and one relation, $b^{-1} = aba^{-1}$. The automorphism group consists of the transformations $a \to ab^n$, $b \to b$. In the usual notation these transformations correspond to $\tau \to \tau + 1$. There is no element of the automorphism group that corresponds to exchanging σ and τ.

Thus the situation is much murkier than it is for closed strings. Of course this was already evident from the fact that SO(32) is picked out in the heterotic string as a consequence of modular invariance whereas in type I theories it is picked out from the string point of view by finiteness and lack of anomalies which require delicate cancellations between different diagrams. Once could certainly study the consistency of orbifold compactifications of open strings by generalizing these calculations. However, in this paper we will proceed more indirectly by demanding cancellation of anomalies in the low-energy field theory in a way that is consistent with the Chan–Paton rules of open string theory. In particular we will study compactification down to six dimensions since in this case anomaly cancellation is strong enough to uniquely determine the new types of string states which must be included along with their associated Chan–Paton factors.

2. Non-orientable strings and Chan–Paton factors. We will first give a very brief review of some salient features of non-orientable strings. Consider first open bosonic strings in flat space. The standard mode expansion of the open string coordinate is given by

$$X^\mu(\sigma, \tau) = x^\mu + p^\mu \tau + \mathrm{i} \sum_{n \neq 0} \alpha_n^\mu \cos n\sigma \exp(-\mathrm{i}n\tau), \tag{2.1}$$

with σ conventionally taken to run from 0 to π. Flipping the string over so that σ now runs from π to 0 is equivalent to replacing α_n by $(-1)^n \alpha_n$. We therefore define an operator f such that $f^{-1}\alpha_n f = (-1)^n \alpha_n$. Demanding that the string be non-orientable is equivalent to keeping only states with $f = 1$. The operator f is usually referred to as the twist operator but to avoid confusion we will refer to it as the flip operator and use the word twist to refer to strings with boundary conditions modified by the presence of a spacetime orbifold. From the path integral point of view this projection simply corresponds to summing over both orientable and non-orientable surfaces. For example the spectrum of the theory is revealed by the one-loop vacuum functional which consists of path integrals over the annulus and the Möbius strip. Note that if the tachyon state is flip even then the massless vector state is flip odd and is thus projected out.

In superstring theory the closed string sector of a theory containing open strings must also be non-orientable in order to be consistent with the underlying $N = 1$ spacetime supersymmetry. The usual closed string mode expansion is

$$X^\mu(\sigma, \tau) = x^\mu + p^\mu \tau + \frac{\mathrm{i}}{2} \sum_{n \neq 0} \alpha_n^\mu \exp[-2\mathrm{i}n(\tau - \sigma)] + \frac{\mathrm{i}}{2} \sum_{n \neq 0} \tilde{\alpha}_n^\mu \exp[-2\mathrm{i}n(\tau + \sigma)], \tag{2.2}$$

so that the flip operator in this sector acts to interchange α_n with $\tilde{\alpha}_n$. Again from a path integral point of view the projection onto flip even states is achieved by including non-orientable surfaces in the sum over surfaces.

The introduction of Chan–Paton factors is analogous to the introduction of non-abelian θ-vacua in two dimensions [4]. In the path integral one associates to each Riemann surface a factor tr_U P $\exp(\int_\Sigma A)$ where the integral runs over the boundary of the surface, P indicates path ordering, and the trace is evaluated in the representation U of some gauge group. For superstrings G must be SO(N) or Sp(N) and U must be the fundamental representation [5,6]. In a hamiltonian framework the Hilbert space must be enlarged from H to $H\otimes V_L\otimes V_R$ where V_L and V_R are $N\times N$ matrices which transform the "charges" on the end of the string. We can now generalize the flip operator by also interchanging V_L and V_R. For SO(N) the result is that physical states alternate at each mass level between the symmetric tensor representation of SO(N) and the antisymmetric tensor or adjoint representation, starting with a massless vector in the anti-symmetric tensor representation. For Sp(N) the representations are interchanged.

3. Z_2 twists. In discussing non-orientable strings on orbifolds it is clear that twisted closed strings which close only up to a symmetry transformation on the spacetime torus must be included to maintain modular invariance for the torus contribution to the path integral. We will now show that there are also new kinds of twisted open strings which are allowed when one considers strings propagating on orbifolds. For simplicity we will deal only with Z_2 twists in this paper. These twisted open strings first appeared in the work of Corrigan and Fairlie [7] in an attempt to define off-shell string amplitudes. They have been discussed more extensively in recent work by Roy and Singh [8].

Recall that the open superstring action S is given by

$$S=-\frac{1}{2\pi}\int_0^{\pi}\mathrm{d}\sigma\int_{-\infty}^{+\infty}\mathrm{d}\tau\big(\partial_\alpha X_\mu\,\partial_\alpha X^\mu+\mathrm{i}\bar{\psi}_\mu\rho_\alpha\,\partial_\alpha\psi^\mu\big). \tag{3.1}$$

where ψ^μ is a two-component fermion field with left-(right-) moving component $\psi^\mu_{1(2)}$. The equations of motion are found by varying S with respect to X^μ and ψ^μ. In order that the boundary contribution to the variation vanish we must have

$$\delta X\cdot\partial_\sigma X|_0^\pi=0, \tag{3.2}$$

and

$$\psi_1\cdot\delta\psi_1-\psi_2\cdot\delta\psi_2|_0^\pi=0. \tag{3.3}$$

If the string is propagating on a flat background, then the boundary conditions must satisfy

$$\partial_\sigma X^\mu(0,\tau)=\partial_\sigma X^\mu(\pi,\tau)=0, \tag{3.4}$$

and

$$\psi_1^\mu(0,\tau)=\psi_2^\mu(0,\tau),\quad \psi_1^\mu(\pi,\tau)=\pm\psi_2^\mu(\pi,\tau), \tag{3.5}$$

so that the theory is Poincaré invariant. The choice of sign in (3.5) corresponds to the Ramond (R) and Neveu–Schwarz (NS) boundary conditions, respectively. With these boundary conditions the mode expansions for bosons are given by (2.1). For fermions they are

$$\psi^\mu(\sigma,\tau)=\begin{pmatrix}\sum_n\psi_n^\mu\exp[-2\mathrm{i}n(\tau-\sigma)]\\ \sum_n\psi_n^\mu\exp[-2\mathrm{i}n(\tau+\sigma)]\end{pmatrix}, \tag{3.6}$$

where n is integer (half-integer) for Ramond (Neveu–Schwarz) boundary conditions in eq. (3.6).

Now consider the string propagating on a Z_2 orbifold. Translation invariance is broken at the fixed points of the Z_2 transformation and we only need to require that the product of the variation of X and $\partial_\sigma X$ vanish. For example we can take $X^I(0,\tau)=x^I$, where x^I is a fixed point of the Z_2 transformation and $\partial_\sigma X^I(\pi,\tau)=0$ and restrict the variations to vanish at $\sigma=0$. Heuristically we can view the lack of momentum conservation for this solution as due to the curvature at the fixed point. Since X^I is identified with $-X^I$, we may take the image of the string and consider the doubled string in flat space. This corresponds to the well-known classical "dumbbell" solution of a relativistic string propagating in flat space with zero net linear momentum.

Solving for $X^I(\sigma,\tau)$, we find

$$X^I(\sigma,\tau)=x^I+\mathrm{i}\sum_{n=Z+1/2}\alpha_n^I\sin n\sigma\exp(-\mathrm{i}n\tau). \tag{3.7}$$

Because $X^I(\sigma,\tau)$ has half-integer moding, we will call strings of this type twisted open strings. Note that just as untwisted closed strings can be viewed as double covers of untwisted open strings, the double covers of these twisted open strings correspond to twisted closed strings since from (3.7) we see that $X^I(\sigma+2\pi,\tau)=-X^I(\sigma,\tau)$. Since the bosonic fields are twisted, the fermionic fields should also be twisted in order to maintain world sheet supersymmetry. The effect of the twist is to switch the Neveu–Schwarz boundary conditions to Ramond and vice versa for the fermion fields corresponding to the twisted orbifold coordinates. One can also have open strings with both ends at a fixed point of the orbifold. These will contribute to the massless spectrum in superstring theories only if both ends sit at the same fixed point. In this case the mode expansion is

$$X^I(\sigma, \tau) = x^I + \mathrm{i}\sum_n \alpha_n^I \sin n\sigma \exp(-\mathrm{i}n\tau). \quad (3.8)$$

These are still open strings since the uncompactified coordinates still obey the usual open string boundary conditions.

These new types of open strings are definitely possible. We now will argue that they must be included for consistency. In addition we will need to understand the Chan–Paton rules for these twisted open strings. We have not yet found a straightforward derivation of these rules so we will proceed indirectly by requiring cancellation of anomalies. However, our results are in accord with the physical picture that the free end of the string does not "see" the fixed point and thus transforms as a vector of SO(32) as usual, while the fixed end "sees" the fixed point at which the curvature and gauge field strength are concentrated and hence has its charge altered.

4. Anomaly cancellation and Chan–Paton rules. Let us consider string compactification on the Z_2 orbifold limit of the four-dimensional K_3 surface. This orbifold is defined by starting with a four-torus with complex coordinates z_1, z_2 with $z_i \sim z_i + 1$, $z_i \sim z_i + \mathrm{i}$, $i = 1, 2$. We then identify points under the Z_2 transformation $\theta: z_i \to -z_i$, which has sixteen fixed points. As a warming-up we will work out the massless spectrum for the Spin(32)/Z_2 heterotic string on this orbifold with the standard embedding of the Z_2 point group in SO(32), breaking the group to SO(28) ⊗ SO(4) ≃ SO(28) ⊗ SU(2) ⊗ SU(2).

It is convenient to use light-cone gauge and the fermionic formulation of the current algebra where the left-movers consist of eight bosonic coordinates and thirty-two internal fermionic coordinates, which transform as the fundamental representation of SO(32). The normal ordering constant is $c = -1$ for the left-moving NS sector and $c = 2$ for the R sector. Let μ designate the transverse uncompactified coordinates, I the compact coordinates, A the SO(28) vector coordinates and a the SO(4) vector coordinates. Clearly oscillators whose coordinates are labelled by μ and A (I and a) commute (anti-commute) with θ.

There are two sectors – the untwisted sector and the sector twisted by θ. First consider the untwisted sector. The physical states are found by projecting the flat-space Hilbert space onto its θ-invariant subspace. The massless modes in the right-moving NS sector are $\psi^{\mu}_{-1/2}|0\rangle_{\mathrm{R}}$ and $\psi^{I}_{-1/2}|0\rangle_{\mathrm{R}}$ where the θ-eigenvalues of these two sets of modes are $\lambda = 1$ for the former and $\lambda = -1$ for the latter. The left-moving NS modes $\lambda = 1$ are $\tilde{\alpha}^{\mu}_{-1}|0\rangle_{\mathrm{L}}$, $\tilde{\psi}^{A}_{-1/2}\tilde{\psi}^{B}_{-1/2}|0\rangle_{\mathrm{L}}$ and $\tilde{\psi}^{a}_{-1/2}\tilde{\psi}^{b}_{-1/2}|0\rangle_{\mathrm{L}}$. The modes with $\lambda = -1$ are $\tilde{\alpha}^{I}_{-1}|0\rangle_{\mathrm{L}}$ and $\tilde{\psi}^{A}_{-1/2}\tilde{\psi}^{a}_{-1/2}|0\rangle_{\mathrm{L}}$.

The right-moving Ramond sector has an eight-dimensional degenerate ground state which we may express in the weight lattice form as $|\pm\frac{1}{2} \pm\frac{1}{2} \pm\frac{1}{2} \pm\frac{1}{2}\rangle_{\mathrm{R}}$, where the number of + signs is even. Each slot corresponds to the eigenvalue of the corresponding element of the Cartan subalgebra. The ψ^{μ}_0 act as raising and lowering operators on the first two slots and the ψ^{I}_0 act as raising and lowering operators on the second two slots. The operator θ acting on these states is $\exp[2\pi\mathrm{i}(J_{67} + J_{89})]$, where J_{IJ} is the generator of rotations in the IJ plane. The ground states invariant under θ are those with opposite signs in the last two slots. In terms of the untwisted four-dimensional subspace there are two right-handed spinors $|\alpha\rangle_{\mathrm{R}}$, and two left-handed spinors $|\alpha'\rangle_{\mathrm{R}}$. The right-handed spinors have $\lambda = 1$ while the left-handed spinors have $\lambda = -1$. The left-moving Ramond sector will not contribute to the massless spectrum.

Combining left- and right-movers, the physical states are the graviton, dilaton and antisymmetric tensor, along with their superpartners, one right-handed gravitino and one left-handed spinor. There are also the gauge bosons and their right-handed superpartners which transform as the **(392, 1, 1)**, **(1, 3, 1)** and **(1, 1, 3)** representations of SO(28) ⊗ SU(2) ⊗ SU(2). In addition there are four left-handed fermions that are gauge singlets and one left-handed fermion that transforms as

(28, 2, 2) along with their scalar superpartners.

Next consider the twisted sector. Twisted NS (R) fermions will now be integer (half-integer) moded. Thus we see that both the right-moving NS and R sectors have four half-integer bosonic and fermionic coordinates and four integer bosonic and fermionic coordinates. Clearly the ground state of both sectors is massless. The zero-modes of the NS sector give its ground state a two-fold degeneracy. The R ground state with even fermion number transforms as a left-handed spinor.

The normal ordering constant for the twisted left-moving NS sector is $c=-\frac{1}{2}$. The four zero-modes lead to a degenerate ground state which transforms as the *spinor* or *spinor′* representation of SO(4), where the *spinor (spinor′)* representation has odd (even) fermion number. Therefore the left-moving massless modes with even fermion number are $\tilde{\psi}^A_{-1/2}\,|spinor\rangle_{\rm L}$ and $\tilde{\alpha}^I_{-1/2}\,|spinor'\rangle_{\rm L}$. Combining left- and right-moving modes and remembering that there are sixteen fixed points, the physical states are the sixteen left-handed fermions and the thirty-two real scalars which transform as **(28, 2, 1)** and the sixty-four left-handed fermions and the 128 real scalars which transform as **(1, 1, 2)**.

Now let us check that this spectrum corresponds to an anomaly-free theory in six dimensions. Six-dimensional gauge anomalies are related to the eight-form $\Omega_8=\mathrm{tr}\;F^4$. For the groups SO($N$) the trace in the adjoint representation, denoted by Tr F^4, is related to the trace in the fundamental representation by

$$\mathrm{Tr}\;F^4=(N-8)\mathrm{tr}\;F^4+3(\mathrm{tr}\;F^2)^2. \tag{4.1}$$

It is only necessary to show that the tr F^4 terms vanish, since the $(\mathrm{tr}\;F^2)^2$ terms may always be eliminated by adding a local counterterm to the action. The SO(4) anomaly can also be cancelled by a counterterm since

$$\mathrm{tr}\;F^4=2\;\mathrm{tr_s}\;F^4=2\left(\mathrm{tr_s}\;F^2\right)^2,$$

where $\mathrm{tr_s}$ is the trace in the spinor representation, which is the fundamental representation of SU(2). Enumerating the SO(28) states, we find one right-handed spinor in the adjoint representation and twenty left-handed spinors in the fundamental representation [‡1]. Ω_8 is

$$\Omega_8=\mathrm{Tr}\;F^4-20\;\mathrm{tr}\;F^4=3(\mathrm{tr}\;F^2)^2.$$

Therefore the SO(28) gauge group is not anomalous.

Six-dimensional theories may also have gravitational anomalies. This anomaly is described by the eight-form

$$\Omega_8^{\rm G}\sim(275-7n)(\mathrm{tr}\;R^2)^2-(980-4n)\;\mathrm{tr}\;R^4, \tag{4.2}$$

where n is the difference between the number of left-handed and right-handed spinors. The second term in the expression cannot be cancelled by a local counterterm in the action. However, if we count the states we find that $n=245$ and the tr R^4 piece does not contribute. The remaining piece may again be cancelled by a local counterterm. Mixed anomalies may be similarly cancelled.

Now let us work out the spectrum of the type I theory on this orbifold keeping in mind the requirements of anomaly cancellation. Again we embed the Z_2 twist in SO(32) by decomposing a Chan–Paton factor in the fundamental of SO(32) into the fundamental representation of the SO(28) subgroup which is invariant when acted upon by θ, and the fundamental representation of SO(4) which changes sign. We can treat the open string coordinates in exactly the same fashion as we did for the right-moving coordinates in the heterotic string.

The untwisted open strings must be projected onto states invariant under f and θ. To accomplish this, we act on the states with the operator $\frac{1}{2}(1+\theta)\frac{1}{2}(1+f)$, which in the path integral formalism is equivalent to having contributions from $\frac{1}{4}[(1)_{\rm A}+(\theta)_{\rm A}+(1)_{\rm MS}+(\theta)_{\rm MS}]$ where $(1)_{\rm A}$ refers to the annulus untwisted along the τ direction, etc. Thus the physical massless states in the NS sector are the SO(28) gauge bosons $\psi^\mu_{-1/2}\,|0\rangle_{[A,B]}$, the SO(4) gauge bosons $\psi^\mu_{-1/2}\,|0\rangle_{[a,b]}$, and the scalar matter fields $\psi^I_{-1/2}\,|0\rangle_{Aa}$. The massless states in

‡1 In performing this counting it is essential to remember that the **(2, 2)** of SU(2)⊗SU(2) is a four-dimensional *real* representation, while the **(2, 1)** is pseudoreal.

the Ramond sector are the right-handed SO(28) gluinos $|\alpha\rangle_{[A,B]}$, the right-handed SO(4) gluinos $|\alpha\rangle_{[a,b]}$, and the two left-handed matter multiplets $|\alpha'\rangle_{Aa}$. Clearly the massless states in the untwisted sector are the same as in the heterotic case since the original massless spectra were the same and we are simply projecting onto group invariant states.

Let us move on to the twisted open strings. Either the $\sigma=0$ or $\sigma=\pi$ end of the string can sit at a fixed point but the non-orientability of the string requires that these states not be considered as physically distinguishable. The twisted open string coordinates are analogous to the twisted right-movers in the heterotic string and have a mode expansion given by (3.7). As we mentioned before, the charge at the fixed end of the string is altered. If we compare these states to the analogous states of the twisted heterotic string, it is clear that in order for the SO(28) anomalies to cancel the free end must have a Chan–Paton factor that transforms in the fundamental representation of SO(32), while the fixed-end Chan–Paton factor transforms as **(1, 2, 1)**. The projection onto θ-invariant states excludes the massless states with SO(4) labels on the free end. Therefore, there are thirty-two real scalars and sixteen left-handed spinors which transform as **(28, 2, 1)**.

Consider next the contribution of open strings with both ends fixed to a fixed point. From eq. (3.8) the oscillators are again integer moded, but with a subtle difference. Both ends are fixed and thus the Möbius strip will contribute to the path integral. Under $\sigma\to\pi-\sigma$, the bosonic oscillators transform as $\alpha_n^I\to(-1)^{n+1}\alpha_n^I$. One sees that the oscillators have an extra sign from the usual untwisted case under this transformation. Again, in order to maintain world sheet supersymmetry, the fermions should also shift by a sign under the flip operation. The consequence of this is that the massless states will be symmetric in the labels at the ends of the string, not antisymmetric as in the untwisted case. Therefore, one concludes that the NS sector has sixty-four massless real scalars transforming in the **(1, 3, 1)** representation. Likewise the Ramond sector has sixteen left-handed spinors transforming in the same representation.

The determination of the closed string states is more straightforward, and perhaps less ad hoc. Since the strings are non-orientable and the Z_2 twist is compatible with the first homology group of the Klein bottle it is necessary to include the Klein bottle diagram in the path integral. To find the spectrum for the closed string, it is sufficient to determine the spectrum for an oriented closed superstring and then symmetrize the right- and left-movers. Carrying this out, we are left with the graviton, dilaton and antisymmetric tensors, along with their superpartners, a right-handed gravitino and a left-handed spinor. We also find in the untwisted sector the fermion states $\frac{1}{2}(\psi^I_{-1/2}|0\rangle_L\otimes|\alpha'\rangle_R+(L\leftrightarrow R)$ with a two-fold degeneracy, along with their superpartners. In the twisted sector both the NS and R states have four-fermion zero-modes and are at the massless level. Therefore we will find sixteen left-handed spinors along with their superpartners. This completes the spectrum. Notice that the spectrum differs slightly from the Spin(32)/Z_2 case. But if we count up the states, we find that the gauge and gravitational anomalies still cancel. So we see that the cancellation of anomalies essentially fixes the Chan–Paton factors for the twisted open string states.

5. Discussion. We have argued that the consistency of Z_2 orbifold compactifications of the type I SO(32) superstring requires the inclusion of "twisted" open strings in the spectrum. In order to make our case completely convincing it would be nice to demonstrate the vanishing of the dilation tadpole at one-loop level. Work on this is in progress.

This work can be easily extended to compactifications down to four dimensions. One can also find an open string solution with the standard embedding on a six-dimensional $G=Z_2\otimes Z_2$ orbifold, where the generators of G are $g=\exp[2\pi i\frac{1}{2}(J_{67}+J_{89})]$ and $g'=\exp[2\pi i\frac{1}{2}(J_{45}+J_{89})]$. The low energy theory is $SO(26)\otimes[U(1)]^3$ where one U(1) is anomalous. (This is also the case for the Spin(32)/Z_2 heterotic string on the same orbifold, although the spectrun is slightly different). The twisted open strings have U(1) charges on the ends of the string, as opposed to SU(2) quantum numbers. Since Tr $F^3=0$ for SO(26) and there are no pure gravitational anomalies, the low-energy theory is anomaly-free (except of course for the anomalous U(1)).

It is also possible to generalize this work to include Z_N twists and possibly also non-abelian twists. There are some subtleties associated with the non-orientability which will be dealt with elsewhere. In the heterotic string there are also consistent orbifolds that correspond to non-standard embeddings of the point group in SO(32). Some of these may even have phenomenological prospects, for example one can obtain four generations of SO(10) via a Z_4 orbifold [9]. We do not know whether there are analogs of non-standard embeddings for the type I′ theory. The examples we have examined do not seem to be consistent with any simple modification of the Chan–Paton rules but there may be some examples that work. It would clearly be nice to have a more fundamental derivation of the modified Chan–Paton rules for twisted open strings.

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