

MAGNETIC MONOPOLES IN NEUTRON STARS*

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Received 6 December 1982
 (Revised 21 September 1983)

The effects of monopole-antimonopole annihilation on a previously reported bound [1] on the product of the galactic flux of grand unified magnetic monopoles and the cross section for monopole catalyzed nucleon decay: $(F_M/\text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1})(\sigma_{\Delta B}/10^{-27} \text{ cm}^2) \leq 10^{-22}$ are examined for several models of neutron star interiors. For neutron stars with superconducting interiors or large internal magnetic fields this bound is unaltered. In the unlikely event that old neutron stars are not superconducting and have internal magnetic fields $B_{\text{int}} \ll 10^8$ Gauss the effects of monopole-antimonopole annihilation relax the bound to $(F_M/\text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1})(\sigma_{\Delta B}/10^{-27} \text{ cm}^2)^2 \leq 10^{-18}$. Magnetic monopoles may also have a significant effect on the structure of the interior magnetic field in neutron stars.

1. Introduction

Grand unified theories predict the existence of magnetic monopoles with masses $m_M \sim 10^{16}$ GeV [2]. Recent model calculations [3] of monopole-fermion scattering have led to the surprising result that the cross section for monopole catalyzed nucleon decay may be as large as a typical strong-interaction cross section: $\sigma_{\Delta B} \simeq 10^{-27} \text{ cm}^2$. In ref. [1] the observational limits on the X-ray flux of old neutron stars were used to place a bound on the product of the number of grand unified monopoles existing in old neutron stars and the cross section for monopole induced nucleon decay: $N_M(\sigma_{\Delta B}/10^{-27} \text{ cm}^2) \leq 3 \times 10^{14}$. By considering the gravitational capture of monopoles by neutron stars this can be translated into a bound on the galactic flux of monopoles

$$(F_M/\text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1})(\sigma_{\Delta B}/10^{-27} \text{ cm}^2) \leq 10^{-22}. \quad (1)$$

This bound assumes that all monopoles captured by the neutron star remain in the neutron star and continue to catalyze nucleon decay. One might worry about effects which could invalidate these assumptions. For example, each monopole will be surrounded by a “fireball” due to the energy released in nucleon decay. If the

* Supported in part by the National Science Foundation grant PHY80-19754.

outward pressure of this fireball exceeds the Fermi pressure of the nucleons then the nucleon density surrounding the monopole and hence the energy released will decrease. Also if N_M is sufficiently large then the monopole density may become large enough that monopole-antimonopole annihilation becomes important and changes the relation between F_M and N_M . Finally, neutron stars are thought to have superconducting interiors as a result of proton-proton pairing. In this case monopoles can only penetrate the interior by moving through pre-existing flux tubes. This process destroys the flux tubes and hence affects the structure of the magnetic field of the neutron star. If this results in fewer flux tubes than monopoles then the trapping of monopoles in flux tubes will enhance the rate of monopole-antimonopole annihilation.

The purpose of this paper is to study the physics of monopoles in neutron stars to determine how the effects described above affect the bound in eq. (1) and how the presence of monopoles will alter the magnetic field in neutron stars and pulsars. We will first review the bound on $F_M \sigma_{AB}$. Sects. 3 and 4 contain a discussion of monopoles in neutron stars with normal and superconducting interiors respectively. The conclusions are presented in sect. 5.

Since the original version of this paper was written, the limits on the monopole flux discussed here have been examined in greater detail and expanded to include limits from catalysis in a variety of astrophysical objects (e.g. nearby pulsars, white dwarfs, Jupiter, earth, etc.) [4–6]. A few comments explaining the relationship of this work to more recent work may be useful.

For objects other than neutron stars the discussion in sect. 3 can be extended to treat monopole-antimonopole annihilation in these objects except that the monopole temperature will probably be comparable to the ambient temperature and must be determined self-consistently in terms of the monopole distribution and luminosity and the resulting temperature distribution. The original neutron star limits have been clarified by including the effects of interstellar absorption and by an improved estimate of the local neutron star density [7]. By considering measured X-ray fluxes of old, nearby pulsars a particularly stringent limit has been derived [4] ($F_M(\sigma_{AB}/10^{-27} \text{ cm}^2) \leq 10^{-28} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$) if the monopoles captured by the progenitor star on the main sequence are included. In order to establish this more stringent limit one must be able to show that a large enough fraction of the captured monopoles survive the formation of the neutron star. The most efficient annihilation mechanism is monopole-antimonopole confinement when the neutron star goes superconducting. A preliminary estimate [8,9] of this effect suggests that enough monopoles do survive to establish this limit. After the neutron star goes superconducting the treatment in sect. 4 is applicable but probably too simplistic. In particular the broadening of flux tubes at the normal-superconducting interface may lead to trapping of monopoles near the corresponding pole of the neutron star [8]. If this happens the magnetic field will not be reduced significantly and monopole-antimonopole annihilation will be unimportant. If monopoles are not trapped then the

subsequent reduction in the magnetic field is in contradiction with theoretical expectations regarding the age and magnetic field of the recently discovered millisecond pulsar unless the number of monopoles is very small. The exact limits will be presented elsewhere [8]. If the neutron star core also contains a pion condensate then the energy per unit length of the flux tubes in the pion superconductor may be large enough to accelerate any monopoles that reach the core to sufficiently large velocities that they can pass through the crust and escape the neutron star at relativistic velocities [8]. If the majority of monopoles are ejected in this fashion then the most stringent flux limits may come from white dwarfs [5]. Finally, although many details of neutron star interiors are uncertain, it is highly implausible that the core should be superconducting due to proton-neutron pairing as suggested in ref. [10]. Although the p-n interaction is attractive, the difference in proton and neutron Fermi energies makes pairing impossible. The monopole-antimonopole annihilation mechanism suggested in ref. [10] is therefore inoperative.

2. A bound on $F_M \sigma_{\Delta B}$

Monopoles in neutron stars will catalyze nucleon decays at a rate $\sim n_{\text{NS}} \langle \sigma_{\Delta B} v \rangle$ per monopole where $n_{\text{NS}} \sim 2 \times 10^{38} \text{ cm}^{-3}$ is the nucleon density, $\sigma_{\Delta B}$ is the cross section for monopole catalyzed nucleon decay and v is the relative velocity of the nucleon and monopole which in our case is just the Fermi velocity of the nucleons $v_F \sim 0.3c$. For low-energy scattering, as would be relevant for monopoles striking the earth, the calculation of $\sigma_{\Delta B}$ is complicated by the interaction between the monopole and the nucleon magnetic dipole moment as well as by strong-interaction effects. However, for incident nucleon energies of order the neutron Fermi energy in a neutron star, $E_F \sim 100 \text{ MeV}$, these effects are small and $\sigma_{\Delta B}$ should be comparable to a typical strong-interaction cross section $\sigma_{\Delta B} \sim 10^{-27} \text{ cm}^2$. Each monopole then catalyzes $\sim 10^{21} (\sigma_{\Delta B}/10^{-27} \text{ cm}^2)$ decays/sec. Since each decay releases $\sim 1 \text{ GeV}$ of energy the total luminosity due to nucleon decay is

$$L = 2.4 \times 10^{18} N_M (\sigma_{\Delta B}/10^{-27} \text{ cm}^2) \text{ erg s}^{-1}. \quad (2)$$

The best limit on the photon luminosity of old neutron stars comes from surveys for serendipitous X-ray sources which are able to see discrete sources with an X-ray luminosity of $L_{\text{dis}}^\gamma = 10^{31} \text{ erg s}^{-1}$ at a distance of 0.1 kpc [11]. These surveys are sensitive to photon energies between 0.1 keV and 4 keV. A neutron star with an X-ray luminosity of L_{dis}^γ has a surface temperature of $T_s \sim (L_{\text{dis}}^\gamma/4\pi R_{\text{NS}}^2 a c)^{1/4} \sim 0.04 \text{ keV}$ where a is the Stefan-Boltzmann constant and $R_{\text{NS}} \sim 10^6 \text{ cm}$ is the radius of the neutron star. For a blackbody spectrum with temperature T_s the peak of the spectrum occurs at an energy of $\sim 0.11 \text{ keV}$ and $\geq 75\%$ of the energy is radiated at energies $> 0.1 \text{ keV}$. These surveys should therefore be able to see neutron stars at 0.1 kpc with a photon luminosity of L_{dis}^γ . Since the average distance between old neutron

stars in the solar neighborhood is expected to be less than 0.1 kpc [12], the dearth of sources in these surveys can only be explained if the X-ray luminosity of old neutron stars is less than L_{dis}^γ .

The luminosity given by eq. (2) will in general be emitted as both photons and neutrinos. For total luminosities less than $10^{33} \text{ erg s}^{-1}$ the luminosity is dominated by photons unless pions condense in the interior of neutron stars in which case a photon luminosity of $10^{31} \text{ erg s}^{-1}$ corresponds to a *total* luminosity of $10^{33} \text{ erg s}^{-1}$ [13]. By demanding that the luminosity given by eq. (2) be less than this total luminosity we find the bound

$$N_M (\sigma_{\Delta B} / 10^{-27} \text{ cm}^2) \leq 3 \times 10^{14}. \quad (3)$$

We must now relate N_M to the incident galactic flux of monopoles. We will ignore for the moment any monopoles present at the formation of the neutron star. Monopoles which strike the surface of a neutron star will lose energy through hadronic interactions with nucleons, through electromagnetic interactions with electrons and nucleons, and possibly through the creation of flux tubes in a superconducting interior. For monopole velocities v_M less than the neutron Fermi velocity $v_F \sim 0.3c$, energy loss through electron encounters will probably dominate with [14]

$$\frac{dE}{dx} \sim \frac{4\pi^2 n_e (eg)^2}{p_e c} \frac{v_M}{c} \sim \frac{v_M}{c} \times 10^{11} \text{ GeV/cm}, \quad (4)$$

where g is the magnetic charge ($g = \frac{137}{2} e$ for unit Dirac charge), $n_e \sim 10^{36} \text{ cm}^{-3}$ is the electron density and $p_e \sim 100 \text{ MeV}/c$ is the Fermi momentum of the electrons. The creation of flux tubes in a superconducting interior causes an energy loss of $dE/dx \sim 10^{10} \text{ GeV/cm}$.

Monopoles in the galaxy will gain an energy of $\sim 10^{10} \text{ GeV}$ from the galactic magnetic field and will be accelerated by the galactic gravitational field to velocities $\sim 10^{-3}c$. The gravitational field of a neutron star will give monopoles which strike the surface of the neutron star a velocity equal to the escape velocity $\sim 0.5c$. Using eq. (4) for the energy loss we find that monopoles with $m_M < 10^{17} \text{ GeV}/c^2$ will be stopped in the neutron star. Heavier monopoles may pass through the neutron star but since they lose 10^{16} GeV of energy in passing through the neutron star they will be gravitationally bound and will continue to lose energy by passing through the neutron star until they are stopped. Only monopoles with an original kinetic energy $\frac{1}{2} m_M c^2 \beta_M^2 \geq 10^{16} \text{ GeV}$ can pass through the neutron star and escape. For $\beta_M = 10^{-3}$ this corresponds to $m_M c^2 \geq 10^{22} \text{ GeV}$.

The number of monopoles captured by a neutron star is

$$N_M^{\text{cap}} = 4\pi F_M A_{\text{cap}} T, \quad (5)$$

where T is the age of the neutron star and the capture area is given by

$$A_{\text{cap}} = \pi R_{\text{NS}}^2 \left(\frac{1 + 2m_{\text{NS}}G/v_{\text{M}}^2 R_{\text{NS}}}{1 - R_{\text{S}}/R_{\text{NS}}} \right) \sim 4 \times 10^5 \pi R_{\text{NS}}^2 \quad (v_{\text{M}} = 10^{-3} c), \quad (6)$$

where R_{S} is the Schwarzschild radius and m_{NS} and R_{NS} are the mass and radius of the neutron star respectively.

The best bound on $F_{\text{M}}\sigma_{\Delta B}$ will come from the oldest neutron stars. The progenitors of neutron stars are thought to be stars with masses of 10–20 solar masses. The lifetime of such objects is quite short ($\sim 10^7$ years) as compared to the age of the galaxy ($\sim 10^{10}$ years). Massive stars such as these which were formed shortly after the formation of the galaxy will thus result in neutron stars with ages comparable to the age of the galaxy. We therefore take $T = 10^{10}$ years as a typical age for old neutron stars.

If all monopoles captured remain in the neutron star then eq. (5) gives

$$N_{\text{M}} \sim 5 \times 10^{36} (F_{\text{M}}/\text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}), \quad (7)$$

for a neutron star of age 10^{10} years. From (3) we thus find

$$F_{\text{M}}(\sigma_{\Delta B}/10^{-27} \text{ cm}^2) \leq 10^{-22} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}. \quad (8)$$

3. Monopoles in neutron stars with normal interiors

The monopole distribution in a neutron star with a normal interior will be determined by the competition between the force of gravity, the gas pressure of the monopoles and the internal magnetic field. Pulsars, which are presumably young neutron stars, are thought to have surface magnetic fields of $B_{\text{s}} \sim 10^{12}$ G. Older neutron stars probably have somewhat smaller fields due to flux diffusion in the crust which can trigger a geometrical rearrangement of macroscopic core regions so as to lower the external magnetic field. The millisecond pulsar, which is thought to be an old, accretion-spun-up neutron star, has a surface field of $\sim 10^8$ G according to current models [16]. The size of the internal magnetic fields is even more uncertain. As we will show, in the absence of a magnetic field the monopoles occupy a sphere of radius $\sim 10^{-2}$ cm at the center of the neutron star. It is therefore the magnetic field at the exact center of the neutron star that governs the monopole distribution. In view of the theoretical uncertainty as to the size of this magnetic field we will consider a range of possibilities. We first discuss the monopole distribution when $B_{\text{int}} = 0$ and then discuss how a central magnetic field will alter this distribution.

In the absence of a magnetic field the monopole and nucleon distributions must satisfy the Tolman-Oppenheimer-Volkoff equation of hydrostatic equilibrium

$$\frac{dP(r)}{dr} = - \frac{G[\rho(r) + P(r)/c^2][M(r) + 4\pi r^3 P(r)/c^2]}{r^2[1 - 2GM(r)/c^2]}, \quad (9)$$

where $P(r)$ and $\rho(r)$ are the pressure and density at radius r and $M(r)$ is the total mass interior to r . Since we will be interested in order of magnitude estimates we will ignore the relativistic corrections and use the newtonian form of (9) for the monopole (M) and nucleon (n) distributions:

$$\frac{dP_{M,n}(r)}{dr} = -\frac{GM(r)\rho_{M,n}(r)}{r^2}, \quad \frac{dM(r)}{dr} = 4\pi r^2(\rho_M(r) + \rho_n(r)). \quad (10)$$

We will assume that the monopoles can be treated as an ideal gas at a temperature T_M . If we ignore catalysis of nucleon decay then the monopoles will come into thermal equilibrium with the nucleons at a temperature $\sim 10^2 - 10^4$ K for old neutron stars. In collisions where the monopole catalyzes nucleon decay the phase space of the decay products is not reduced by the filled nucleon Fermi sea so that the monopole sees the ~ 100 MeV Fermi energy of the nucleons. In addition the monopoles will tend to thermalize with the decay products with energies of order a few hundred MeV. T_M will be determined by the competition between cooling due to scattering without nucleon decay and heating due to catalysis of nucleon decay. For a catalysis cross section $\sigma_{\Delta B} \sim 10^{-27}$ cm², $kT_M \sim 200$ MeV is probably a reasonable estimate. In what follows we will explicitly keep track of the dependence on T_M and m_M .

We now want to solve for the monopole density distribution. For $\rho_M \leq \rho_n$, P_n and ρ_n will be essentially constant over the volume occupied by the monopoles. Writing $P_M(r) = kT_M \rho_M(r)/m_M$ and differentiating (10) with respect to r yields

$$\frac{1}{r^2} \frac{d}{dr} \left(\frac{r^2}{\rho_M} \frac{d\rho_M}{dr} \right) = -\frac{4\pi G m_M}{kT_M} (\rho_M(r) + \rho_n(r)). \quad (11)$$

Changing variables to $\rho_M(r) = \rho_M(0)\exp(-W_M(r))$ and setting $r = \bar{r}x \equiv (kT_M/4\pi G m_M \rho_M(0))^{1/2} x$ gives

$$\frac{d^2}{dx^2} W_M(x) + \frac{2}{x} \frac{d}{dx} W_M(x) = e^{-W_M(x)} + \rho_n/\rho_M(0). \quad (12)$$

The boundary conditions are $W_M(0) = 0$, so that $\rho_M(0)$ is the central density, and $W'_M(0) = 0$ which follows from spherical symmetry.

For $\rho_n/\rho_M(0) \rightarrow 0$, eq. (12) is the isothermal form of the Lane-Emden equation [16]. For the case we are interested in, $\rho_n/\rho_M(0) \gg 1$,

$$\rho_M(r) = \rho_M(0) \exp\left(-\frac{\rho_n}{6\rho_M(0)} \frac{r^2}{\bar{r}^2}\right), \quad (13)$$

with $\rho_M(0) = (m_M N_M / \pi^{3/2} r_M^3)$ where r_M is given by

$$\begin{aligned} r_M &= \bar{r} \left(\frac{6\rho_M(0)}{\rho_n} \right)^{1/2} = \left(\frac{3kT_M}{2\pi G m_M \rho_n} \right)^{1/2} \\ &= 3 \times 10^{-2} \left(\frac{kT_M}{200 \text{ MeV}} \right)^{1/2} \left(\frac{10^{16} \text{ GeV}}{m_M c^2} \right)^{1/2} \text{ cm}. \end{aligned} \quad (14)$$

This solution is valid as long as $\rho_M(0) < \rho_n$ or

$$N_M < N_M^c = 4 \times 10^{18} \left(\frac{kT_M}{200 \text{ MeV}} \right)^{3/2} \left(\frac{10^{16} \text{ GeV}}{m_M c^2} \right)^{5/2}. \quad (15)$$

If $N_M > N_M^c$ then (12) must be integrated numerically to find $\rho_M(r)$. Since N_M^c monopoles would yield a luminosity of $\sim 10^{36} (\sigma_{AB}/10^{-27} \text{ cm}^2) \text{ erg s}^{-1}$ and since we will show that monopole-antimonopole annihilation is already appreciable for $N_M < N_M^c$, we will assume from now on that $N_M < N_M^c$ so that we can use the distribution (13).

Given the monopole distribution for $B = 0$ let us consider the effects which may alter the bound (1). The first is the creation of a fireball surrounding each monopole and the monopole distribution. If the pressure from the fireball becomes larger than the Fermi pressure the nucleon density around the monopole(s) will decrease and hence the rate of catalysis of nucleon decay will decrease. The Fermi pressure at the center must support the neutron star against gravitational collapse. The central pressure is thus

$$P_c \sim \frac{GM_{NS}^2}{4R_{NS}^4} \sim \frac{h^2}{15m_p \pi^2} \left(\frac{3\pi^2 \rho_{NS}}{m_p} \right)^{5/3} \sim 5 \times 10^{34} \text{ dynes cm}^{-2}. \quad (16)$$

Each monopole releases $\sim 10^{18} (\sigma_{AB}/10^{-27} \text{ cm}^2) \text{ erg s}^{-1}$ in a sphere of radius $\sim r_p \sim 10^{-13} \text{ cm}$. A rough estimate of the pressure due to the outgoing decay products is

$$P_d \sim \frac{10^{18} (\sigma_{AB}/10^{-27} \text{ cm}^2) \text{ erg s}^{-1}}{4\pi (10^{-13} \text{ cm})^2 c} \sim 3 \times 10^{32} (\sigma_{AB}/10^{-27} \text{ cm}^2) \text{ dynes cm}^{-2}. \quad (17)$$

Therefore if $\sigma_{AB} \leq 10^{-25} \text{ cm}^2$ the energy output per monopole will be limited to $\sim 10^{20} \text{ erg s}^{-1}$ due to the decrease in the surrounding nucleon density. The pressure

due to the decay products of the total monopole distribution is

$$P_d^{\text{tot}} \sim \frac{N_M \times 10^{18} \text{ erg s}^{-1} (\sigma_{\Delta B} / 10^{-27} \text{ cm}^2)}{4\pi r_M^2 c} \sim 3 \times 10^9 N_M \times \left(\frac{\sigma_{\Delta B}}{10^{-27} \text{ cm}^2} \right) \left(\frac{m_M}{10^{16} \text{ GeV}} \right) \left(\frac{200 \text{ MeV}}{kT_M} \right), \quad (18)$$

which should be negligible for $N_M \leq N_M^c$.

Monopole-antimonopole annihilation can also alter the bound (1) at sufficiently large monopole densities. Monopole-antimonopole bound states with bound state energies $\sim kT_M$ can be formed through two- or three-body monopole encounters or through scattering of monopoles off of the surrounding nucleons in the presence of an antimonopole. These bound states have a very large principal quantum number determined by

$$I_n = \frac{1}{4} m_M c^2 \frac{\alpha_g^2}{n^2} = kT_M \approx 200 \text{ MeV}, \quad (19)$$

where $\alpha_g = g^2/\hbar c$. We thus have $n \approx 4 \times 10^9$. Since the orbital frequency $W_n \sim I_n/\hbar n$ is much less than the plasma frequency $\omega_p \approx (4\pi n_e e^2/m_e)^{1/2} \approx 50 \text{ MeV}/\hbar$ these states cannot spiral down and decay by the usual emission of dipole radiation. The lifetime of these bound states is difficult to estimate. They can lose energy through ohmic losses in the plasma or through collisions with nucleons and free monopoles. However they may also gain energy through nucleon collisions leading to nucleon decay although the cross section for this process in the presence of a monopole-antimonopole bound state probably decreases significantly once the bound state radius is less than the QCD confinement scale. In light of this uncertainty we will make the worst case assumption that the lifetime is much less than the other relevant time scales so that the capture rate and the annihilation rate may be taken as equal.

A classical calculation of monopole-antimonopole capture through emission of radiation gives a capture cross section [17]

$$\sigma_c \sim \alpha_g^2 \left(\frac{\hbar}{m_M c} \right)^2 \left(\frac{m_M c^2}{kT_M} \right)^{7/5} \quad (20)$$

Dicus, Page and Teplitz [18] have reexamined this calculation and find that photon discreteness effects modify σ_c for monopole velocities $v_M/c \ll \alpha_g^{-5/4} \sim 10^{-2}$. In this regime capture almost always occurs through emission of a photon with $\hbar\omega > kT_{Mn}$

$\sim 200 \text{ MeV} > \hbar\omega_p$ so that plasma effects are small. They find a capture cross section of

$$\begin{aligned}\sigma_c^Q &\sim 2^7 3^{-3/2} \pi \alpha_g^3 \left(\frac{\hbar}{m_M c} \right)^2 \left(\frac{c}{v_M} \right)^2 \ln(\alpha_g^{-5/4} c/v_M) \\ &\sim 5 \times 10^{-36} \left(\frac{10^{16} \text{ GeV}}{m_M c^2} \right) \left(\frac{200 \text{ MeV}}{kT_M} \right) \text{ cm}^2.\end{aligned}\quad (21)$$

They have also discussed the formation of bound states due to three-body collisional recombination. Using their results we find the ratio of the time scales for three-body and two-body capture to be

$$\tau_2/\tau_3 \sim 6 \left(\frac{m_M}{10^{16} \text{ GeV}} \right) \left(\frac{n_M}{2 \times 10^{22} \text{ cm}^{-3}} \right). \quad (22)$$

Since N_M^c monopoles corresponds to a central density of

$$n_M(0) = \frac{N_M^c}{\pi^{3/2} r_M^3} \sim 2 \times 10^{22} \left(\frac{200 \text{ MeV}}{kT_M} \right)^{3/2} \left(\frac{m_M c^2}{10^{16} \text{ GeV}} \right)^{3/2} \text{ cm}^{-3}, \quad (23)$$

two-body capture should dominate for $N_M \ll N_M^c$.

If two-body annihilation dominates then the number of monopoles in the neutron star obeys the differential equation

$$\frac{dN_M}{dt} = 4\pi F_M A_{\text{cap}} - \langle v_M \sigma_c^Q \rangle \int_0^\infty 4\pi r^2 n_M^2(r) dr. \quad (24)$$

Assuming $N_M(0) = 0$, this has the solution

$$N_M(t) = N_M^{\text{eq}} \tanh(t/t_0), \quad (25)$$

where

$$\begin{aligned}N_M^{\text{eq}} &= \left[4\pi F_M A_{\text{cap}} / (\langle v_M \sigma_c^Q \rangle / r_M^3 \pi^{3/2}) \right]^{1/2} \\ &\simeq 2 \times 10^{24} (F_M / \text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1})^{1/2} \left(\frac{kT_M}{200 \text{ MeV}} \right), \\ t_0 &= \left[4\pi F_M A_{\text{cap}} (\langle v_M \sigma_c^Q \rangle / r_M^3 \pi^{3/2}) \right]^{-1/2} \\ &\simeq 4 \times 10^{-3} (F_M / \text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1})^{-1/2} \left(\frac{kT_M}{200 \text{ MeV}} \right) \text{ y}.\end{aligned}\quad (26)$$

For $F_M \geq 10^{-26} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$, $10^{16} \text{ y}/t_0 \gg 1$ so that the number of monopoles at $t = 10^{10}$ years is given by N_M^{eq} . The bound on $N_M \sigma_{\Delta B}$ then gives

$$(F_M/\text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1})^{1/2} (\sigma_{\Delta B}/10^{-27} \text{ cm}^2) (kT_M/200 \text{ MeV}) \leq 10^{-9}, \quad (27)$$

which is much less stringent than (1). For example a Parker flux [19] of $F_M \sim 10^{-16} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ would be consistent with this bound if either $\sigma_{\Delta B}$ or kT_M is a factor of 10 less than our estimates or if the annihilation cross section is enhanced due to scattering of monopoles off of nucleons in the presence of antimonopoles.

This calculation of monopole-antimonopole annihilation assumes the magnetic field to be zero over the range of the monopole distribution. This is probably incorrect even for old neutron stars. While old neutron stars with small interior magnetic fields may exist, the estimate of the number density of old neutron stars in the solar neighborhood was derived from the estimated birth rate of pulsars in the solar neighborhood. Pulsars presumably *do* have large magnetic fields $\sim 10^{12} \text{ G}$ and even if their interiors are not superconducting fairly large fields should survive for longer than 10^{10} years [20].

The presence of a uniform magnetic field near the center of a neutron star [$(\nabla B/B)^{-1} \gg r_M$] will separate the monopole and antimonopole distributions and hence reduce the annihilation rate. For large magnetic fields we can approximate the monopole and antimonopole distributions as point charges each carrying a total charge of gN_M . The force due to the magnetic field must balance the gravitational attraction towards the center giving

$$Bg = \frac{4}{3} \pi \rho_{\text{NS}} m_M G s_M, \quad (28)$$

where $2s_M$ is the distance between the monopole and antimonopole distribution. For $B = 10^{12} \text{ G}$ we find

$$s_M \sim 4 \times 10^4 (10^{16} \text{ GeV}/m_M c^2) \text{ cm} \gg r_M, \quad (29)$$

so that annihilation should be completely negligible. For large N_M and/or small B the Coulomb attraction between the monopole and antimonopole distributions will become important and we can no longer treat the distributions as point charges. This will occur for

$$B \lesssim \left(\frac{N_M}{g} \right)^{1/3} \left(\frac{4}{3} \pi \rho_{\text{NS}} G m_M \right)^{2/3} \sim 3 \times 10^2 N_M^{1/3} (m_M c^2/10^{16} \text{ GeV})^{2/3} \text{ G}. \quad (30)$$

Thus an internal magnetic field of $\sim 10^8$ Gauss should be sufficient to eliminate monopole-antimonopole annihilation and preserve the bound (1) unless the monopoles have masses much greater than the Planck mass.

4. Monopoles in neutron stars with superconducting interiors

In this section we will assume that the interior of a typical neutron star consists of an inner sphere of radius 5–8 km which is superconducting due to proton-proton pairing, and an outer normal region of thickness 1–3 km. The superconducting region is characterized by two lengths, the proton coherence length $\xi_p \sim (2/\pi k_F)(E_p/\Delta_p)$ where $E_p = \hbar^2 k_F^2/2m_p$, k_F is the proton Fermi wave number and Δ_p is the energy gap; and the penetration length λ_p which is given by the London formula for $\lambda_p \gg \xi_p$, $\lambda_p \sim m_p c^2/4\pi n_p e^2$. For typical neutron star parameters $\lambda_p \sim 10^{-11}$ cm and $\xi_p \sim 10^{-12}$ cm so that $\xi_p/\lambda_p < \sqrt{2}$ (type (II) superconductivity). Any magnetic field present will permeate this superconducting region as a series of vortices or flux tubes of radius λ_p , each carrying one flux quantum $\phi_0 = hc/2e \approx 2 \times 10^{-7}$ G cm². The critical fields between which this vortex state is stable are $H_{c1} \sim (\phi_0/4\pi)\lambda_p^2 \sim 10^{15}$ G and $H_{c2} \sim (\phi_0/2\pi)\xi_p^2 \sim 3 \times 10^{16}$ G. Since the magnetic fields in pulsars are thought to be $\sim 10^{12}$ G the superconducting interior will exhibit the Meissner effect and expel the magnetic field. However, due to the enormous electrical conductivity in the normal state ($\sigma > 10^{29}$ s⁻¹) the time it takes to expel the magnetic field is $\geq 10^{13}$ years [20] which is much greater than the age of the galaxy. Old neutron stars should therefore have $B \sim 10^{12}$ G fields which permeate the superconducting interior as a series of flux tubes with area density $\sim B/\phi_0 \sim 5 \times 10^{18}(B/10^{12} \text{ G}) \text{ cm}^{-2}$. A typical neutron star will thus be permeated by $\sim \pi R_{\text{NS}}^2 B/\phi_0 \sim 10^{31}(B/10^{12} \text{ G})$ flux tubes.

Monopoles which strike a neutron star will lose most of their energy in traversing the normal crust of thickness 1–3 km. Their velocity upon encountering the superconducting interior will be determined by their gravitational acceleration and their energy loss through electron encounters. Using eq. (4) for the energy loss gives a terminal velocity of $v_M/c \sim 4 \times 10^{-3}$. A monopole which penetrates the superconducting interior will have its magnetic flux squeezed into a flux tube connecting the monopole to the surface. A monopole with unit Dirac charge carries two units of flux ($4\pi g = 2\phi_0$) and for a type (II) superconductor will emanate two flux tubes each carrying one flux quantum. The interior field in a flux tube is $H_1 \sim \phi_0/\pi\lambda_p^2 \sim 10^{15}$ G and each flux tube has an energy per unit length $\sim (H_1^2/8\pi)\pi\lambda_p^2 \sim 10^{10}$ GeV/cm. The increase in energy due to flux tube formation is greater than the gain in gravitational potential energy for monopoles with mass less than 10^{17} GeV so these monopoles will be suspended at the surface of the superconducting core by the Meissner effect. The monopole will follow the magnetic field lines near the surface of the core. These field lines will eventually penetrate the core in the form of a flux tube. The monopole can then drop down the flux tube and since the monopole carries two units of flux, the magnetic field emanating from the flux tube will be opposite to that of its neighbors as shown in fig. 1. The monopole will then fall towards the center of the neutron star along the line of the flux tube. The flux tube containing the monopole will be attracted to adjacent flux tubes since the magnetic

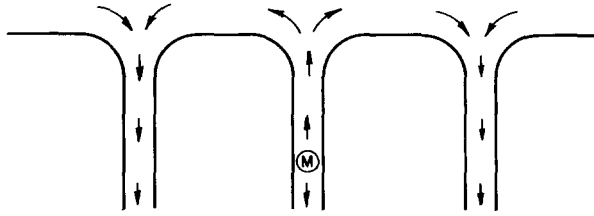


Fig. 1. Flux tube configuration after swallowing monopole.

fields are oppositely oriented. Motions which bring the flux tube within a distance λ_p of its neighbors will result in the pinching off of the flux tubes and the formation of loops of flux tube as shown in fig. 2.

Moving flux tubes are subject to a viscous drag force per unit length $f_r = \eta v$ where v is the flux tube velocity [21]. In pure laboratory superconductors eddy current losses to the degenerate core electrons gives $\eta \sim \phi_0 H_{c2} \sigma_n / c^2$ where H_{c2} is the upper critical field and $\sigma_n = n_e e^2 \tau_r / m^*$ is the electrical conductivity in the normal state. m^* is the effective mass, $p/m^* = 2E_p / \partial p$, n_e is the electron number density, and τ_r is the electron relaxation time. Since in neutron stars the electron mean free path is $\gg \lambda_p$ and we are considering the motion of only a single flux tube, the relaxation time should be replaced by the transit time for an electron through the flux tube $\sim \lambda_p / v_F$. For a core density of $10^{15} \text{ g cm}^{-3}$ we expect $n_e \sim 4 \times 10^{37} \text{ cm}^{-3}$ giving $\eta \sim 3 \times 10^{10}$.

The pinch off of the flux tube containing the monopole can be caused by attraction of adjacent flux tubes at the normal-superconducting interface, by thermal motions of the flux tube, or by the motion of the flux tube-monopole system due to the gravitational force on the monopole. The first process cannot be estimated reliably while the second is slow due to the drag force and low temperature. The time scale for pinch off due to the gravitational force on the monopole can be estimated as follows. The position of the flux tube obeys a diffusion equation

$$\eta \frac{\partial U}{\partial t} - T \frac{\partial^2 U}{\partial x^2} = F_g \delta(x), \quad (31)$$

where U and x are defined in fig. 2, $T \sim 10^7 \text{ erg cm}^{-1}$ is the tension in the flux tube and F_g is the gravitational force on the monopole. The solution gives $U(0, t) = F_g \eta^{-3/2} (Tt/\pi)^{1/2}$. The flux tube must on average move a mean free path $\sim d_f^2 / \lambda_p$ before encountering another flux tube. This takes a time $t_p \sim 3 \times 10^{-4} (B_{12} \rho_{15} r_6 m_{-8})^{-2} \text{ sec}$ where the core density is $\rho_{15} \times 10^{15} \text{ g cm}^{-3}$, the distance of the monopole from the center of the neutron star is $r_6 \times 10^6 \text{ cm}$ and the monopole mass is $m_{-8} \times 10^{-8} \text{ g}$.

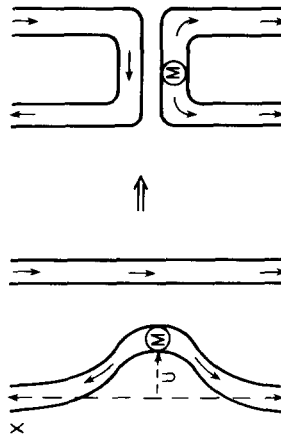


Fig. 2. Flux tube pinch off.

Once the flux tube pinches off the resulting loop of flux tube will work its way through the neutron star, pulling the monopole with it. The velocity of the flux loop is determined by $2T = \eta v_{\text{loop}} d_f$ so that it takes a time $t_\ell \sim 0.6 R_6 B_{12}^{-1/2}$ sec for the loop to move a distance of the core radius $\sim R_6 \times 10^6$ cm and thus reach the surface of the superconducting core. When the monopole reaches the surface it will return along the surface field lines until it again encounters an attractive flux tube and will then repeat the above scenario. The monopole velocity during the return trip is determined by equating the magnetic force Bg with the energy loss $\sim v/c \times 10^{11}$ GeV/cm. This gives a return time $t_r = 0.1 B_{12}^{-1}$ s. Each monopole thus destroys $2(t_p + t_{\ell c} + t_r)^{-1}$ flux tubes per second with a concomitant decrease in the magnetic field.

For reasonable values of N_m this will not result in fewer flux tubes than monopoles within the age of the galaxy so monopole-antimonopole annihilation is unimportant. However it can result in an appreciable reduction of the core magnetic field for numbers of monopoles comparable to the bound in (3). Possible observational implications of the reduction of the core field will be discussed in ref. [8].

5. Conclusions

Our main conclusion is that the bound on $F_M \sigma_{AB}$ found in ref. [1] is in fact correct based on our present theoretical knowledge of neutron star physics. If for some unknown reason old neutron stars have normal interiors with small internal magnetic fields, then it is possible to reconcile observation of old neutron star X-ray luminosities with a galactic flux of monopoles comparable to the Parker flux $F_M \sim 10^{16} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$. Flux limits based on younger neutron stars [4] are however insensitive to monopole-antimonopole annihilation. Magnetic monopoles may play an important role in the reduction of neutron star magnetic fields.

I would particularly like to thank E. Kolb and S. Colgate for discussions and correspondence that clarified the issues discussed here and J. Sauls for discussions of superconductivity in neutron stars. I am also grateful to M. Ruderman and J. Shaham for pointing out a mistake in the original version of this manuscript.

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