

STRING THEORIES IN TEN DIMENSIONS WITHOUT SPACETIME SUPERSYMMETRY

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We analyze the possibilities for new modular invariant ten-dimensional string theories without spacetime supersymmetry which are twisted versions of the closed superstring or of the heterotic string. The most interesting possibility is a tachyon-free theory with gauge group $O(16) \otimes O(16)$, chiral fermions, and a positive one-loop contribution to the cosmological constant.

The anomaly cancellation mechanism discovered by Green and Schwarz [1] and the construction of the heterotic string [2] have prompted many discussions of the relationship between various string theories as well as the possibility of new types of string theories [3,4]. It is tempting to interpret the presence of a tachyonic state in the twenty-six dimensional bosonic string as a signal of vacuum instability and hence to search for a stable vacuum which would be tachyon-free. It has been proposed [4] that ten-dimensional superstrings might emerge as the correct vacuum in such an approach. Although there are many intriguing relations between the bosonic string and superstring theories, no compelling dynamical mechanism has yet appeared.

The presence of a tachyon in the bosonic string, as well as divergences associated with dilaton tadpoles [5], are indications of vacuum instability at either the classical or quantum level, but they do not indicate any fundamental inconsistency in the theory. It would be interesting to know whether there are any other string theories with the same degree of consistency as the twenty-six dimensional closed bosonic string; that is, Lorentz invariant, modular invariant, interacting string theories. In this letter we will present several such theories. Although they are related in a rather simple fashion to previously known theories, they do not seem to have been discussed previously.

The first model we will consider is related to the ten-dimensional closed superstring. We will work in light-cone gauge and with the old Ramond-Neveu-Schwarz

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formalism for the spinning string. The physical degrees of freedom of this theory are the transverse bosonic coordinates $X^i(\sigma, \tau)$, $i = 1, \dots, 8$ transforming as the $\mathbf{8}_c$ representation of $\text{SO}(8)$, with the mode expansion

$$X^i(\sigma, \tau) = x^i + p^i \tau + \frac{1}{2}i \sum_{n \neq 0} \frac{\alpha_n^i}{n} e^{-2in(\tau - \sigma)} + \frac{1}{2}i \sum_{n \neq 0} \frac{\tilde{\alpha}_n^i}{n} e^{-2in(\tau + \sigma)}, \quad (1)$$

and the fermionic coordinates $\psi^i(\tau - \sigma)$ and $\tilde{\psi}^i(\tau + \sigma)$, also transforming as vectors under $\text{SO}(8)$. Consider first the (NS, NS) sector with antiperiodic boundary conditions for both left- and right-moving fermionic fields. The mode expansions for ψ^i and $\tilde{\psi}^i$ are then

$$\begin{aligned} \psi^i(\tau - \sigma) &= \sum_{n \in Z + 1/2} b_n^i e^{-2in(\tau - \sigma)}, \\ \tilde{\psi}^i(\tau + \sigma) &= \sum_{n \in Z + 1/2} \tilde{b}_n^i e^{-2in(\tau + \sigma)}. \end{aligned} \quad (2)$$

The (mass)² operator, in units where $\alpha' = \frac{1}{2}$, is given by

$$\frac{1}{4}(\text{mass})^2 = (L_0 - \frac{1}{2}) + (\bar{L}_0 - \frac{1}{2}), \quad (3)$$

where

$$\begin{aligned} L_0 &= \sum_{n=1}^{\infty} \alpha_n^i \alpha_n^i + \sum_{n=1/2}^{\infty} n b_n^i b_n^i, \\ \bar{L}_0 &= \sum_{n=1}^{\infty} \tilde{\alpha}_n^i \tilde{\alpha}_n^i + \sum_{n=1/2}^{\infty} n \tilde{b}_n^i \tilde{b}_n^i, \end{aligned} \quad (4)$$

and physical states are subject to the constraint $L_0 = \bar{L}_0$ which follows from invariance under rigid shifts in σ . The $-\frac{1}{2}$ terms added to L_0 and $\bar{L}_0$ in the (mass)² operator arise from normal ordering and are required for closure of the Lorentz algebra in light-cone gauge.

As it stands this theory is inconsistent due to a lack of modular invariance. To see this consider the string path integral continued to euclidean space $(\sigma, t) = (\sigma, i\tau)$ for a world sheet with the topology of a torus. The metric on the torus, modulo conformal equivalence, depends on a single complex modular parameter τ with $\text{Im } \tau > 0$. The modular group is generated by the transformations

$$\begin{aligned} \tau &\rightarrow \tau + 1, \\ \tau &\rightarrow -\frac{1}{\tau}, \end{aligned} \quad (5)$$

which correspond respectively to cutting the torus along a line of constant t and reconnecting with a 2π twist, and to interchanging σ and t directions. The

Neveu-Schwarz model corresponds to (σ, t) boundary conditions of $(-1, -1)$ on both sets of fermionic fields ψ' and $\tilde{\psi}'$. This model is clearly invariant under $\tau \rightarrow -1/\tau$, but not under $\tau \rightarrow \tau + 1$, which takes $(-1, -1)$ to $(-1, +1)$. This can easily be checked by explicit calculation. For example, the one-loop contribution to the cosmological constant in this theory is easily calculated along the lines of [6, 7] to be

$$\begin{aligned} \Lambda_{1\text{-loop}} &\sim - \int \frac{d^2\tau}{(\text{Im } \tau)^2} (\text{Im } \tau)^{-4} \left| \frac{\prod_{n=1}^{\infty} (1 + z^{n-1/2})}{\prod_{n=1}^{\infty} (1 - z^n)} \right|^{16} \\ &\sim - \int \frac{d^2\tau}{(\text{Im } \tau)^2} (\text{Im } \tau)^{-4} \left| \frac{\vartheta_3(0, \tau)}{\vartheta_1'(0, \tau)} \right|^8, \end{aligned} \quad (6)$$

where $z = e^{2\pi i \tau}$ and the Jacobi theta functions ϑ_i are described in [8]. Using the transformation properties of the theta functions one has

$$\begin{aligned} \left| \frac{\vartheta_3(0, \tau + 1)}{\vartheta_1'(0, \tau + 1)} \right|^8 &= \left| \frac{\vartheta_4(0, \tau)}{\vartheta_1'(0, \tau)} \right|^8, \\ \left| \frac{\vartheta_3(0, -1/\tau)}{\vartheta_1'(0, -1/\tau)} \right|^8 &= |\tau|^8 \left| \frac{\vartheta_3(0, \tau)}{\vartheta_1'(0, \tau)} \right|^8, \\ (\text{Im}(-1/\tau))^{-4} &= |\tau|^8 (\text{Im } \tau)^{-4}, \end{aligned} \quad (7)$$

so the integrand is invariant under $\tau \rightarrow -1/\tau$ but not under $\tau \rightarrow \tau + 1$.

In order to ensure modular invariance it is necessary to sum over a modular invariant combination of boundary conditions on the fermionic fields ψ' and $\tilde{\psi}'$. The usual closed superstring corresponds to summing over the boundary conditions $(-1, -1)$, $(+1, -1)$, $(-1, +1)$ and $(+1, +1)$ independently for ψ' and $\tilde{\psi}'$. In terms of a hamiltonian formulation, the sum over t boundary conditions for fields antiperiodic in σ corresponds to the GSO projection [9] onto states with $G = (-1)^F = 1$ and $\tilde{G} = (-1)^{\tilde{F}} = 1$, where $F = \sum_{n=1/2}^{\infty} b'_{-n} b'_n - 1$ and $\tilde{F} = \sum_{n=1/2}^{\infty} \tilde{b}'_{-n} \tilde{b}'_n - 1$ are the world-sheet fermion numbers for ψ' and $\tilde{\psi}'$. The -1 in the definition of F , or equivalently the assignment of fermion number -1 to the tachyonic vacuum, is required for modular invariance. The GSO projection in the (NS, NS) sector destroys invariance under $\tau \rightarrow -1/\tau$. To restore this invariance it is necessary to include "twisted" sectors where the string closes only up to an element of the $Z_2 \otimes Z_2$ group with elements $\{1, (-1)^F, (-1)^{\tilde{F}}, (-1)^{F+\tilde{F}}\}$. These are just the (NS, NS), (R, NS), (NS, R), and (R, R) sectors of the closed string, where in

the Ramond (R) sectors the fermion fields are periodic in σ with the mode expansions

$$\begin{aligned}\psi^I &= \sum_{m \in \mathbb{Z}} d_m^I e^{-2im(\tau + \sigma)}, \\ \tilde{\psi}^I &= \sum_{m \in \mathbb{Z}} \tilde{d}_m^I e^{-2im(\tau - \sigma)}.\end{aligned}\quad (8)$$

The operators L_0 and $\bar{L}_0$ in the Ramond sectors are given by

$$\begin{aligned}L_0 &= \sum_{m=1}^{\infty} \alpha_{-m}^I \alpha_m^I + \sum_{m=1}^{\infty} m d_{-m}^I d_m^I, \\ \bar{L}_0 &= \sum_{m=1}^{\infty} \tilde{\alpha}_{-m}^I \tilde{\alpha}_m^I + \sum_{m=1}^{\infty} m \tilde{d}_{-m}^I \tilde{d}_m^I.\end{aligned}\quad (9)$$

The normal-ordering constant, which enters into the (mass)² operator and into the constraint, is zero for the Ramond sectors. In these sectors $(-1)^F$ and $(-1)^{\tilde{F}}$, which must anticommute with right-moving and left-moving fermionic fields respectively, are given by

$$\begin{aligned}(-1)^F &= \pm d_0^1 \dots d_0^8 (-1)^{\sum_{m=1}^{\infty} d_{-m}^I d_m^I}, \\ (-1)^{\tilde{F}} &= \pm \tilde{d}_0^1 \dots \tilde{d}_0^8 (-1)^{\sum_{m=1}^{\infty} \tilde{d}_{-m}^I \tilde{d}_m^I}.\end{aligned}\quad (10)$$

Choosing the same sign in the definition of $(-1)^F$ and $(-1)^{\tilde{F}}$ in the Ramond sector yields the chiral type IIB superstring, with the massless fields in the (R, R) sector transforming as the $\text{SO}(8)$ representations contained in $\mathbf{8}_v \otimes \mathbf{8}_v$. Choosing the opposite sign yields the type IIA superstring, with the massless fields in the (R, R) sector transforming as $\mathbf{8}_v \otimes \mathbf{8}_s$.

The closed string theory originally considered by GSO [9] and recently reconsidered in [10, 11] actually consists only of the (NS, NS) and (NS, R) sectors. While this gives a supersymmetric spectrum, since the right-movers alone are supersymmetric, it does not result in a modular invariant theory. This is clear from the (σ, τ) boundary conditions and can also be checked by an explicit one-loop calculation [11].

There is one further modular invariant combination of boundary conditions, which consists of summing over the *same* boundary conditions for left-movers and right-movers. That is, we project the (NS, NS) sector onto states with $(-1)^{F+\tilde{F}} = 1$ and add the "twisted" (R, R) sector, again projected onto states with $(-1)^{F+\tilde{F}} = 1$. There are actually two such theories, corresponding to the relative sign in the definitions of $(-1)^F$ and $(-1)^{\tilde{F}}$ in the (R, R) sector, just as in the case of the closed superstring. Each theory contains only spacetime bosons. However, the bosonic states are not a subset of the bosonic states of the superstring, since we are now

including states with $(-1)^F = (-1)^{\tilde{F}} = -1$ which were previously discarded. In particular, the tachyon in the (NS, NS) sector is now physical. The partition function for the two theories is the same. In the projected (NS, NS) sector it is given by

$$\frac{|A(z)|^2 + |B(z)|^2}{2|z||f(z)|^{16}}. \quad (11)$$

Here we have defined the functions

$$\begin{aligned} A(z) &\equiv \prod_{n=1}^{\infty} (1 + z^{n-1/2})^8 = \vartheta_3^4/f(z)^4, \\ B(z) &\equiv \prod_{n=1}^{\infty} (1 - z^{n-1/2})^8 = \vartheta_4^4/f(z)^4, \\ C(z) &\equiv 8 \prod_{n=1}^{\infty} (1 + z^n)^8 = \vartheta_2^4/(2z^{1/2}f(z)^4), \\ f(z)^3 &\equiv \prod_{n=1}^{\infty} (1 - z^n)^3 = \vartheta_1'/2z^{1/8}, \end{aligned} \quad (12)$$

using the infinite-product representations of the Jacobi theta functions and the standard abbreviation $\vartheta_i \equiv \vartheta_i(0, \tau)$. The partition function for the projected (R, R) sector is given by

$$\frac{|C(z)|^2}{2|f(z)|^{16}}. \quad (13)$$

Adding (11) to (13) and using (12) we find the total partition function to be

$$\frac{2^7(|\vartheta_2|^8 + |\vartheta_3|^8 + |\vartheta_4|^8)}{|\vartheta_1'|^8}, \quad (14)$$

which when combined with a factor $(\text{Im } \tau)^{-4}$ from the integration measure is clearly invariant under modular transformations.

So far our analysis has relied on the old Ramond-Neveu-Schwarz formalism. Given the equivalence between this formalism and the manifestly supersymmetric Green-Schwarz formalism, it is natural to ask whether this theory can be obtained within the latter formalism. In this formalism the fields ψ' and $\tilde{\psi}'$ are replaced by periodic fields S^a and $\tilde{S}^a$ transforming as the spinor $\mathbf{8}_s$ of $\text{SO}(8)$ (or as the $\mathbf{8}_s$ and $\mathbf{8}_c$ in the case of the type IIA superstring). The irreducible representation of the S_0^a zero mode algebra is sixteen-dimensional and consists of an $\mathbf{8}_c$ plus an $\mathbf{8}_s$ of $\text{SO}(8)$. The $\tilde{S}_0^a$ algebra is treated similarly. The only symmetry of this theory which leaves

invariant ten-dimensional Minkowski space is a 2π spatial rotation, say $e^{2\pi i J_{12}}$, or in other words $(-1)^{F_S}$, where F_S is spacetime (as opposed to world-sheet) fermion number. Projecting onto states with $(-1)^{F_S} = 1$ removes all the fermionic states and leaves all the bosonic states of the superstring. To preserve modular invariance we must include the twisted sector where the string closes only up to a transformation by $(-1)^{F_S}$. In other words, we must include the sector where S^a and $\tilde{S}^a$ are both antiperiodic, projected onto states with $(-1)^{F_S} = 1$. In this sector the normal-ordering addition to L_0 and $\tilde{L}_0$ is $-\frac{1}{2}$ for the same reason as it was in the (NS, NS) sector above. So the lowest state is a singlet tachyon with $\frac{1}{4}(\text{mass})^2 = -1$, the next states consist of

$$\tilde{S}_{-1/2}^a |0\rangle_L \otimes S_{-1/2}^b |0\rangle_R$$

and are massless, and so on. It is easy to see that this procedure simply reproduces the previous spectrum of states.

So far we have only discussed the free spectrum of this theory. Lorentz invariance and the inclusion of interactions follow in a straightforward way, since in the Ramond-Neveu-Schwarz formalism we have only changed the projection relating the left- and right-hand sectors which were previously consistent. Because this theory has a tachyon and no fermions it is unlikely to be of much interest except as a toy model and we will not discuss it further.

It is natural to ask whether there are any analogous new theories which can be constructed from the heterotic string. Rather than looking at various combinations of $(-1)^F$ projections in the fermionic formulation of the heterotic string, we for the moment use the Green-Schwarz formalism for the right-moving fermionic degrees of freedom and the bosonic Frenkel-Kač construction of the $E_8 \otimes E_8$ or $\text{Spin}(32)/Z_2$ affine Lie algebra. We then need to consider the possible twists that leave invariant ten-dimensional Minkowski space.

One possibility is to again twist by a 2π spatial rotation. However, in this case it is easy to check that one simply recovers the heterotic string, with the spacetime fermions now appearing in the twisted sector. We can also twist by an $E_8 \otimes E_8$ or $\text{Spin}(32)/Z_2$ gauge transformation, but it can be shown [12] that this can only give back one or the other heterotic string. (For certain twists it is possible to start with $E_8 \otimes E_8$ and obtain $\text{Spin}(32)/Z_2$ and vice versa.) The only new possibility is to accompany a 2π spatial rotation with an order two twist of the $E_8 \otimes E_8$ or $\text{Spin}(32)/Z_2$ affine Lie algebra.

We thus consider Z_2 twists of the form

$$e^{2\pi i J_{12}}|_{\text{spacetime}} \cdot \gamma_\delta, \quad (15)$$

where γ_δ is of order two acting in $E_8 \otimes E_8$ or $\text{Spin}(32)/Z_2$. We can choose γ_δ to lie in a Cartan subalgebra, in which case we can represent γ_δ as a translation (or "shift") by a vector $\pi\delta$ on the $E_8 \otimes E_8$ or $\text{Spin}(32)/Z_2$ maximal torus; i.e. $\gamma_\delta = \exp(2\pi i p^I \delta^I)$, where p^I is the center of mass momentum operator for the string

coordinate X^I on the maximal torus. Since $\gamma_\delta^2 = 1$, 2δ must be a lattice vector. In the twisted sector the string closes only up to a 2π spatial rotation plus a translation of the coordinates of the $E_8 \otimes E_8$ or $\text{Spin}(32)/Z_2$ torus by $\pi\delta$: i.e. $X^I(\pi) = X^I(0) + \pi\delta^I$. Thus in the twisted sector the momenta p^I lie on a "shifted" lattice: $p \in L + \delta$, where $L = \Gamma_8 \otimes \Gamma'_8$ or Γ_{16} , Γ_8 is the root lattice of E_8 , and Γ_{16} is the weight lattice of $\text{Spin}(32)/Z_2$. Hence the left-moving contribution to the $(\text{mass})^2$ operator is shifted by $\frac{1}{2}\delta^2$. In order to have matching contributions to the $(\text{mass})^2$ from left- and right-movers, as required by modular invariance [12], δ^2 must be an integer. Also, shift vectors δ which differ by a lattice vector are clearly equivalent, so we can reduce the length of δ by subtracting off lattice vectors. It turns out [13] that by this procedure we can arrange that $\delta^2 \leq 2$ in the case of Γ_{16} , and $\delta = (\delta_1, \delta'_1)$ with $\delta_1^2 \leq 1$, $\delta'^2_1 \leq 1$ in the case of $\Gamma_8 \otimes \Gamma'_8$. We thus conclude that $\delta^2 = 0, 1$ or 2 .

If $\delta^2 = 0$, then γ_δ acts as the identity and we just get back the heterotic string. For $\delta^2 = 1$ there are two inequivalent choices for δ for $E_8 \otimes E_8$ and three for $\text{Spin}(32)/Z_2$. (Shift vectors which are related by some automorphism of the lattice correspond to conjugate, and hence equivalent, gauge transformations.) A convenient choice for the roots of E_8 is given by

$$\begin{aligned} \pm e_i \pm e_j, & \quad i \neq j, \\ \frac{1}{2}(\pm e_1 \pm e_2 \pm \cdots \pm e_8) & \quad (\text{even number of } + \text{ signs}), \end{aligned} \quad (16)$$

where the e_i , $i = 1, \dots, 8$, are orthonormal vectors. The roots of $\text{SO}(32)$ are given by $\pm e_i \pm e_j$, $i \neq j$, where the e_i , $i = 1, \dots, 16$, are orthonormal vectors. The weight lattice of $\text{Spin}(32)/Z_2$ is generated by integer combinations of these roots as well as one of the spinor weights of $\text{Spin}(32)$, say $\frac{1}{2}(e_1 + \cdots + e_{16})$. Using these bases, and using powers to denote repeated entries, the inequivalent shift vectors δ of length one are given by

$$\begin{aligned} \delta &= (1, 0^7; 0^8), \quad \left(\frac{1}{2}, \frac{1}{2}, 0^6; \frac{1}{2}, \frac{1}{2}, 0^6\right), \quad E_8 \otimes E_8, \\ \delta &= (1, 0^{15}), \quad \left(\left(\frac{1}{2}\right)^4, 0^{12}\right), \quad \left(\left(\frac{1}{4}\right)^{16}\right), \quad \text{Spin}(32)/Z_2. \end{aligned} \quad (17)$$

For $\delta^2 = 2$ there is a unique choice for either $E_8 \otimes E_8$ or $\text{Spin}(32)/Z_2$, given by

$$\begin{aligned} \delta &= (1, 0^7; 1, 0^7), \quad E_8 \otimes E_8, \\ \delta &= \left(\left(\frac{1}{2}\right)^8, 0^8\right), \quad \text{Spin}(32)/Z_2. \end{aligned} \quad (18)$$

The theories with $\delta^2 = 1$ all have physical tachyons occurring in the twisted sector. (Recall that the right-handed vacuum is at $-\frac{1}{2}$ in this sector, which matches the left-handed vacuum at $-1 + \frac{1}{2}\delta^2 = -\frac{1}{2}$.) Note that the tachyons are not gauge singlets. We will not analyze these theories in much detail. However, we should at

least determine the spectrum of massless states to verify that they correspond to anomaly-free ten-dimensional theories. In each case we find the bosonic part of the ten-dimensional supergravity multiplet, $g_{\mu\nu}, B_{\mu\nu}, \phi$, coming from the untwisted sector. In addition we find the following massless states. For $\delta = (1, 0^7; 0^8)$ in $E_8 \otimes E_8$: gauge bosons in the adjoint representation of $O(16) \otimes E_8$ and fermions in the $\mathbf{8}_v$ of $SO(8)$ and the $(\mathbf{128}, \mathbf{1})$ of $O(16) \otimes E_8$ from the untwisted sector, and fermions in the $\mathbf{8}_v$ of $SO(8)$ and the $(\mathbf{128}', \mathbf{1})$ of $O(16) \otimes E_8$ from the twisted sector. For $\delta = (\frac{1}{2}, \frac{1}{2}, 0^6; \frac{1}{2}, \frac{1}{2}, 0^6)$ in $E_8 \otimes E_8$: gauge bosons in the adjoint representation of $E_7 \otimes SU(2) \otimes E_7 \otimes SU(2)$ and $\mathbf{8}_v$ fermions in the $(\mathbf{56}, \mathbf{2}; \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}; \mathbf{56}, \mathbf{2})$ representation of the gauge group from the untwisted sector, and $\mathbf{8}_v$ fermions in the $(\mathbf{56}, \mathbf{1}; \mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{2}; \mathbf{56}, \mathbf{1})$ representation from the twisted sector. For $\delta = (1, 0^{15})$ in $Spin(32)/Z_2$: gauge bosons in the adjoint representation of $SO(32)$ and no massless fermions. For $\delta = ((\frac{1}{2})^4, 0^{12})$ in $Spin(32)/Z_2$: gauge bosons in the adjoint representation of $SO(24) \otimes SO(8)$ and $\mathbf{8}_v$ fermions in the $(\mathbf{24}, \mathbf{8}_v)$ representation of the gauge group from the untwisted sector, and $\mathbf{8}_v$ fermions in the $(\mathbf{24}, \mathbf{8}_v')$ representation of the gauge group from the twisted sector. For $\delta = ((\frac{1}{2})^{16})$ in $Spin(32)/Z_2$: gauge bosons in the adjoint representation of $SU(16) \otimes U(1)$ and $\mathbf{8}_v$ fermions in the $(\mathbf{120}, +\mathbf{2}) \oplus (\mathbf{120}, -\mathbf{2})$ representation of the gauge group from the untwisted sector, and $\mathbf{8}_v$ fermions in the $(\mathbf{120}, -\mathbf{2}) \oplus (\mathbf{120}, +\mathbf{2})$ representation from the twisted sector. All of these theories are anomaly-free, either because they are non-chiral as in the third case, or because the anomalies cancel trivially using the relations $\text{Tr}_{\mathbf{8}_v} F'^{2n} = \text{Tr}_{\mathbf{8}_v} F'^{2n} = \text{Tr}_{\mathbf{8}_v} F'^{2n}$ for $SO(8)$ and $\text{Tr}_{\mathbf{128}} F'^{2n} = \text{Tr}_{\mathbf{128}} F'^{2n}$ for $SO(16)$, or using the Green-Schwarz anomaly cancellation mechanism [1] for $(E_7 \otimes SU(2))^2$ and for $SU(16) \otimes U(1)$. There is one other possible set of Z_2 twists, which cannot be represented by shift vectors because they involve the outer automorphism of $E_8 \otimes E_8$, which exchanges the two E_8 's. These cases also can be shown to have physical tachyons, and we will not discuss them further here.

Finally we come to the most interesting case which is $\delta^2 = 2$. This case leads to a tachyon-free theory since in the twisted sector the right-handed vacuum at $-\frac{1}{2}$ does not match the left-handed vacuum at $-1 + \delta^2 = 0$. So in the untwisted sector the $L_{-1} = \bar{L}_{-1}$ constraint projects out the tachyon from the bosonic string, since the superstring vacuum is at level zero; whereas in the twisted sector this constraint projects out the tachyon from the twisted superstring, since the twisted bosonic string vacuum is at level zero! It turns out that starting with either $E_8 \otimes E_8$ or $Spin(32)/Z_2$ yields the same theory, so we will just analyze the theory starting from $E_8 \otimes E_8$. In addition, although the spectrum is not hard to work out in this "shifted" bosonic formulation, it is even simpler in the fermionic formulation of $E_8 \otimes E_8$. In this formulation the shift by $\pi\delta$ in $E_8 \otimes E_8$ is equivalent to a 2π rotation in each $O(16)$ factor in the $O(16) \otimes O(16)'$ subgroup of $E_8 \otimes E_8$. (See ref. [12] for details.) So the twist (15) becomes

$$R = e^{2\pi i J_{12}}|_{\text{spacetime}} e^{2\pi i J_{12}} e^{2\pi i J'_{12}}, \quad (19)$$

where j_{12} and j'_{12} are generators of rotations in $O(16)$ and $O(16)'$ respectively.

In the fermionic formulation of $E_8 \otimes E_8$ the fermions χ^I and χ'^I transform as $(16, 1') \oplus (1, 16')$ under $O(16) \otimes O(16)'$, so in the twisted sector the fermions obey the same boundary conditions as before. However, the fermion number of the vacuum changes by 1 for each set of fermions, so the $(-1)^F$ and $(-1)^{F'}$ projections are changed; i.e. in the twisted sector we keep the states which previously had $(-1)^F = (-1)^{F'} = -1$. Among the left-moving states with $\frac{1}{4}(\text{mass})^2 \leq 0$ this leaves only the massless states $\chi^I_{-1/2} \chi'^J_{-1/2} |0\rangle_L$, from the (NS, NS) sector. (States in the (R, R) sector start at $\frac{1}{4}(\text{mass})^2 = 1$; states in the (NS, R) and (R, NS) sectors start at $\frac{1}{4}(\text{mass})^2 = 0$, but these states have the wrong eigenvalue for either $(-1)^F$ or $(-1)^{F'}$.) To form physical states we must now combine left- and right-moving states at the same mass level and project onto $R = R_L R_R = 1$, where $R_{L(R)}$ is the left(right)-moving contribution to R . In the untwisted sector, those states with $R_L = 1$ will combine with the bosonic right-moving states ($R_R = 1$) to form physical bosons; those states with $R_L = -1$ will combine with the fermionic right-moving states ($R_R = -1$) to form physical fermions. In the twisted sector, the eigenvalues of R_L and R_R are a bit peculiar: bosonic right-moving states have $R_R = -1$ and fermionic right-moving states have $R_R = 1$, and the contributions to R_L from the tensor and spinor representations of $O(16)$ and $O(16)'$ are similarly reversed. Putting these results together, we find that the massless states consist of the bosonic part of the supergravity multiplet $(g_{\mu\nu}, B_{\mu\nu}, \phi)$, gauge bosons in the adjoint representation of $O(16) \otimes O(16)'$, 8_s fermions in the $(128, 1) \oplus (1, 128)$ representation of the gauge group, and 8_s fermions in the $(16, 16)$ representation.

We will show below that this string theory is modular invariant. One then expects that the low-energy ten-dimensional field theory should be anomaly free. To see that this is the case first note that pure gravitational anomalies cancel because there are 256 8_s and 256 8_s fermions, albeit in different representations of $O(16) \otimes O(16)$. To check the cancellation of pure gauge anomalies and mixed anomalies, we need to relate traces in the spinor 128 representation of $O(16)$ to those in the vector 16:

$$\begin{aligned} \text{Tr}_{128} F^6 &= 16 \text{tr} F^6 + \frac{15}{4} (\text{tr} F^2)^3 - 15 \text{tr} F^2 \text{tr} F^4, \\ \text{Tr}_{128} F^4 &= 6 (\text{tr} F^2)^2 - 8 \text{tr} F^4, \\ \text{Tr}_{128} F^2 &= 16 \text{tr} F^2. \end{aligned} \tag{20}$$

Here we denote traces in the vector 16 representation simply by tr . We will also denote by Tr the signed trace over the fermionic representations which are present at the massless level, $\text{Tr} \equiv \text{Tr}_{(128, 1)} + \text{Tr}_{(1, 128)} - \text{Tr}_{(16, 16)}$. The trace relations (20) may be derived from trace relations for the adjoint representation of E_8 , along with relations between the vector and adjoint traces for $O(16)$ (both sets of which may be found in ref. [1]), by decomposing the adjoint of E_8 into the adjoint plus spinor of

O(16). Using (20), we can write the twelve-form [14] representing the pure gauge anomaly as

$$\frac{1}{15} \text{Tr } F^6 = \frac{1}{4} \left[(\text{tr } F_1^2)^3 + (\text{tr } F_2^2)^3 \right] - (\text{tr } F_1^2 + \text{tr } F_2^2)(\text{tr } F_1^4 + \text{tr } F_2^4), \quad (21)$$

and the twelve-forms for the mixed anomalies as

$$\frac{1}{24} \text{Tr } F^4 \text{tr } R^2 = \left\{ \frac{1}{4} \left[(\text{tr } F_1^2)^2 + (\text{tr } F_2^2)^2 - \text{tr } F_1^2 \text{tr } F_2^2 \right] - \text{tr } F_1^4 - \text{tr } F_2^4 \right\} \text{tr } R^2, \quad (22)$$

$$\frac{1}{30} \text{Tr } F^2 \left[\frac{1}{8} \text{tr } R^4 + \frac{5}{32} (\text{tr } R^2)^2 \right] = 0. \quad (23)$$

Here F_1 and F_2 are field strengths in the two O(16) factors, $\text{tr } R^{2n}$ is a trace in the vector representation of SO(9, 1), and the relative normalization of the three terms is correct. Note the absence of $\text{tr } F_i^6$ terms, which makes it possible to cancel the anomalies with local counterterms. From (21), (22) and (23) we can extract the actual anomalies along the lines of ref. [1]; they are given by the expression

$$\begin{aligned} G_f \sim \int \bigg\{ & \frac{1}{4} \left[\omega_{2Y,1}^1 (\text{tr } F_1^2)^2 + \omega_{2Y,2}^1 (\text{tr } F_2^2)^2 \right] - \frac{1}{4} (\omega_{2Y,1}^1 + \omega_{2Y,2}^1) (\text{tr } F_1^4 + \text{tr } F_2^4) \\ & - \frac{1}{4} (\omega_{6Y,1}^1 + \omega_{6Y,2}^1) (\text{tr } F_1^2 + \text{tr } F_2^2) \\ & + \left[\frac{1}{6} (\omega_{2Y,1}^1 \text{tr } F_1^2 + \omega_{2Y,2}^1 \text{tr } F_2^2) - \frac{1}{12} (\omega_{2Y,1}^1 \text{tr } F_2^2 + \omega_{2Y,2}^1 \text{tr } F_1^2) \right. \\ & \quad \left. - \frac{1}{4} (\omega_{6Y,1}^1 + \omega_{6Y,2}^1) \right] \text{tr } R^2 \\ & \left. + \frac{1}{24} \left[\frac{1}{4} \left[(\text{tr } F_1^2)^2 + (\text{tr } F_2^2)^2 - \text{tr } F_1^2 \text{tr } F_2^2 \right] - \text{tr } F_1^4 - \text{tr } F_2^4 \right] \right\}. \quad (24) \end{aligned}$$

The $2n$ -forms $\omega_{2nY(L)}^1$ can be defined via the gauge variations of the Yang-Mills(Lorentz) Chern-Simons forms, $\delta \omega_{(2n+1)Y(L)}^0 = -d\omega_{2nY(L)}^1$. The Chern-Simons forms can in turn be defined via $d\omega_{(2n+1)Y}^0 = \text{tr } F^{n+1}$, $d\omega_{(2n+1)L}^0 = \text{tr } R^{n+1}$. Taking the antisymmetric tensor field $B_{\mu\nu}$ to transform as

$$\delta B = -\xi (\omega_{2Y,1}^1 + \omega_{2Y,2}^1) - \omega_{2L}^1 \quad (25)$$

(modulo exact forms), with $\xi = 1$, the gauge variation of the counterterm

$$\begin{aligned} S_{\text{ct}} \sim \int \bigg\{ & B \left[\frac{1}{4} \left[(\text{tr } F_1^2)^2 + (\text{tr } F_2^2)^2 - \text{tr } F_1^2 \text{tr } F_2^2 \right] - \text{tr } F_1^4 - \text{tr } F_2^4 \right] \\ & - \frac{1}{4} (\omega_{3Y,1}^0 + \omega_{3Y,2}^0) (\omega_{7Y,1}^0 + \omega_{7Y,2}^0) - \frac{1}{4} \omega_{3Y,1}^0 \omega_{3Y,2}^0 (\text{tr } F_1^2 - \text{tr } F_2^2) \\ & + \omega_{3L}^0 \left[\frac{1}{6} (\omega_{3Y,1}^0 \text{tr } F_1^2 + \omega_{3Y,2}^0 \text{tr } F_2^2) - \frac{1}{12} (\omega_{3Y,1}^0 \text{tr } F_2^2 + \omega_{3Y,2}^0 \text{tr } F_1^2) \right. \\ & \quad \left. - \frac{1}{4} (\omega_{7Y,1}^0 + \omega_{7Y,2}^0) \right] \bigg\} \quad (26) \end{aligned}$$

cancels precisely the anomalous gauge variation (24) due to the fermions.

Finally we wish to show that this theory is in fact modular invariant. There are three sets of fermionic fields in the heterotic string, the spacetime fermions ψ' and the $O(16) \otimes O(16)'$ fermions χ' and χ'' , each with either NS or R boundary conditions. One can check that this $O(16) \otimes O(16)'$ theory has in the untwisted sector the following four triples of (spacetime, $O(16)$, $O(16)'$) boundary conditions: (NS, NS, NS), (NS, R, R), (R, NS, R), and (R, R, NS), all with the usual $(-1)^F$ projections. In addition the twisted sector contains the remaining four triples: (R, R, R), (R, NS, NS), (NS, R, NS), and (NS, NS, R), all with the *opposite* set of states surviving the $(-1)^F$ projections (because the fermion number of each of the three Fock space vacua has changed by 1). With this fact in hand it is straightforward to evaluate the partition functions for the spacetime bosons and for the spacetime fermions. Using the definitions (12) they are given respectively by

$$P_B(z, \bar{z}) = \frac{1}{2z^{1/2}|f(z)|^{16}} \left\{ [A(z) - B(z)] \left[4\bar{z}C(\bar{z})^4 + \frac{1}{4\bar{z}}(A(\bar{z})^2 + B(\bar{z})^2)^2 \right] \right. \\ \left. + [A(z) + B(z)] \left[2C(\bar{z})^2(A(\bar{z})^2 - B(\bar{z})^2) \right] \right\}, \quad (27)$$

$$P_F(z, \bar{z}) = \frac{C(z)}{|f(z)|^{16}} \left\{ 2C(\bar{z})^2[A(\bar{z})^2 + B(\bar{z})^2] + 4\bar{z}C(\bar{z})^4 \right. \\ \left. + \frac{1}{4\bar{z}}[A(\bar{z})^2 - B(\bar{z})^2]^2 \right\}. \quad (28)$$

Use of the identities $\vartheta_2^4 + \vartheta_4^4 = \vartheta_3^4$ and $\vartheta_1' = \vartheta_2\vartheta_3\vartheta_4$ results in considerable simplification.

We can evaluate for example the one-loop contribution to the cosmological constant which is given [6, 7] by

$$\Lambda_{1\text{-loop}} \sim - \int_F \frac{d^2\tau}{(\text{Im } \tau)^2} (\text{Im } \tau)^{-4} [P_B(z, \bar{z}) - P_F(z, \bar{z})], \quad (29)$$

with $z = e^{2\pi i \tau}$ and the integration region restricted to a fundamental domain F of the modular group, e.g. $-\frac{1}{2} \leq \text{Re } \tau \leq \frac{1}{2}$, $|\tau| \geq 1$, $\text{Im } \tau > 0$. Using (27), (28) and (12)

we find

$$\Lambda_{1\text{-loop}} \sim - \int_{\mathbb{F}} \frac{d^2\tau}{(\text{Im } \tau)^2} \frac{1}{(\text{Im } \tau)^4 \vartheta_1'^4(\tau)} \left\{ \frac{\vartheta_2^4(\tau)}{\vartheta_2^8(\bar{\tau})} + \frac{\vartheta_4^4(\tau)}{\vartheta_4^8(\bar{\tau})} - \frac{\vartheta_3^4(\tau)}{\vartheta_3^8(\bar{\tau})} \right\}. \quad (30)$$

The integrand is clearly invariant under modular transformations. What is not so clear is the sign or magnitude of $\Lambda_{1\text{-loop}}$. Although we know of no simple method for evaluating integrals of this type, the absence of a tachyon ensures the absence of divergences in the integral, and the existence of convergent power series expansions for the theta functions throughout the fundamental domain makes numerical evaluation of the integral straightforward but tedious. We have found that in fact (30) is *positive*, in contrast to the negative contribution in the bosonic string [7] or in the models analyzed by Rohm [6].

This theory appears to be the unique modular-invariant, tachyon-free string theory in ten dimensions without spacetime supersymmetry. At the one-loop level the presence of a nonzero cosmological constant will lead to a dilaton tadpole and hence to quantum instability. It is not clear whether one can shift the vacuum to find a perturbatively stable vacuum also without spacetime supersymmetry, but the prospects appear brighter than for the bosonic string, given the absence of a tachyon in the free spectrum.

It should also be noted that the ten-dimensional theories considered here bear a close resemblance to the models considered by Rohm [6], in which propagation of the closed superstring and also of the heterotic string on a "twisted torus" was studied. A twisted torus results from identifying points in euclidean space under a group containing not just pure translations (which would yield an ordinary torus), but also elements which are combined translations and rotations. If we compactify the first theory we have considered here (which is purely bosonic) on a cubic six-torus and let the lengths of the sides of the torus approach zero (relative to $\sqrt{\alpha'}$), we find that the spectrum approaches that for the closed superstring on a twisted torus (with the twist $\theta = 2\pi$ as defined in [6]), in the limit of vanishing size for the twisted torus. In the case of the heterotic string on a twisted torus, we can also accompany the above-mentioned combined translations and rotations by gauge transformations. If we choose one of the Z_2 gauge transformations which we have listed above, we can find a similar correspondence of the spectrum for the heterotic string on a twisted torus with the spectrum for one of the above ten-dimensional theories compactified on an ordinary torus, again in the limit of vanishing size for the two tori.

Finally, we have also found tachyon-free theories with some of the spacetime dimensions compactified via orbifolds in a way that breaks spacetime supersymmetry. These theories are classically stable in the orbifold limit. If it is possible to go away from the orbifold limit by blowing up the orbifold singularities, then there

should exist conformally invariant nonlinear sigma models with non-Kähler target spaces. These theories, as well as the numerical evaluation of $\Lambda_{1-\text{loop}}$ for the above theory, will be discussed elsewhere [15].

Some of the ten-dimensional tachyonic theories described here have also been considered by N. Seiberg and E. Witten [16]. Related results have been obtained by C. Vafa [16].

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Note added in proof. The possibility of a consistent bosonic string theory in ten-dimensions was suggested by C. Thorn at the November 1985 Santa Fe meeting on string theory. The tachyon-free $O(16) \otimes O(16)$ model has also been discussed by Alvarez-Gaumé, Ginsparg, Moore, and Vafa [17]. Kawai, Lewellen, and Tye [18] have given a classification of modular invariant ten-dimensional strings using a fermionic representation. Their results agree with ours and include a tachyonic E_8 model which is the unique model obtained using the outer automorphism of $E_8 \otimes E_8$ mentioned in the text.

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