

COSMOLOGICAL PRODUCTION OF KALUZA–KLEIN MONOPOLES

Jeffrey A. HARVEY ^a, Edward W. KOLB ^b and Malcolm J. PERRY ^{a,c}

^a *Department of Physics, Princeton University, Princeton, NJ 08544, USA*

^b *Theoretical Astrophysics, Fermi National Accelerator Laboratory, Batavia, IL 60510, USA*

^c *Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge CB3 9EW, UK*

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The cosmological production of Kaluza–Klein monopoles is discussed. The present monopole-to-entropy ratio is calculated in some simple models with the conclusion that this ratio is unacceptably large unless additional mechanisms for entropy production or monopole annihilation are present.

In 1974, 't Hooft [1] and Polyakov [2] demonstrated that the field equations of many spontaneously-broken gauge theories admit magnetic monopoles. Suppose that one starts with some compact gauge group G which is spontaneously broken down to $H \times U(1)_{EM}$, then such a theory will admit soliton-type solutions which correspond to massive, stable magnetic monopoles. The typical mass scale for such objects is M_x/e where M_x is the mass of the heavy vector particles, and e is the coupling constant. These monopoles can be regarded as topological defects in the orientation, in group space, of the vacuum expectation value of the Higgs field. In standard big-bang cosmologies, one starts off with a very hot initial phase ($T \gg M_x/e$). The Higgs field then varies randomly over space due to thermal fluctuations. As the universe cools, the phase where the VEV of the Higgs field does not vanish becomes stable, and the Higgs field becomes frozen. By chance, this field will contain topological defects which will persist to the present day, and manifest themselves as magnetic monopoles [4–6].

It has recently been shown explicitly by Gross and Perry [7] and Sorkin [8] that five-dimensional Kaluza–Klein theories [9,10], which are unified theories of electromagnetism and gravitation, admit the existence of magnetic-monopole solutions. These solutions are completely non-singular. In general, in any Kaluza–Klein theory, one has an n -dimensional space which is locally the product of some compact d -dimensional

space Σ with an isometry group I , with a four-dimensional spacetime M . In general the Σ -valued fibres over M need not be trivial. The result is that the action of I can have isolated fixed points. In Kaluza–Klein theories I is interpreted as a low-energy gauge group [11]. If I is of the form $H \times U(1)$, then these fixed points can be magnetic monopoles. Typically, one expects the size of Σ to be of the order of the Planck scale. The mass of the monopoles is then of the order of M_{Planck}/e where e is now the low-energy coupling constant of the effective low-energy I -valued gauge fields. This precisely parallels the scenario of spontaneously-broken gauge theory models. Gravitation in n dimensions has a gauge symmetry Diff^n the diffeomorphism group in n dimensions (group of coordinate transformations). This group is spontaneously broken down to $\text{Diff}^4 \times I$. Topological defects in the fibration of Σ over M correspond to magnetic monopoles, and possibly other types of excitations [12].

In what follows, we wish to discuss the role of Kaluza–Klein monopoles in cosmology. However, we face a problem. At the present epoch, we know that if the universe really has more than four dimensions, the $(n - 4)$ remaining dimensions must have scale sizes of the order of the Planck length. Furthermore, these scales must be constant over cosmological time scales. Consider a Kaluza–Klein theory in which electromagnetism arises via the Kaluza–Klein mechanism. Then the $U(1)$ symmetry is associated with an S^1 generated

by a Killing vector which commutes with all other Killing vectors contained in I. The "Kaluza-Klein" circle has radius $R \sim 1/e$. Thus e determines the size of the Kaluza-Klein circle. There are (at least) three observations which tell us the e has been constant over cosmological timescales. The first is an observation by Bahcall and Schmidt [13]. They showed that by looking at various emission lines of oxygen in active radio galaxies at redshifts of $z \sim 0.2$, that

$$e(z \sim 0.2)/e(\text{lab}) = 1.0005 \pm 0.001.$$

The second is an observation due to Shlyakhter [14]. Approximately 2×10^9 years ago, a natural nuclear reactor at Oklo in the Gabon operated. Examination of the isotope ratios in the fission products enabled him to deduce that

$$e(\text{reactor})/e(\text{lab}) \simeq 1 \pm 10^{-8}.$$

Finally, even at the era of nucleosynthesis e cannot have been substantially different from at present. The reason is that the neutron-proton mass difference is almost entirely electromagnetic in origin, and it is this quantity which essentially determines the primordial helium and deuterium abundances. These are correctly determined by traditional schemes. The point is that even as far back as a few minutes after the big-bang, the compact part of the spacetime must be substantially the same as now.

This leads us to a problem of how to control the size of the compact piece, and how to arrange some dimensions to be compact. It could be that we should assert that even as far back as the big-bang, part of space was always compact. In this case, the Kaluza-Klein scenario arises as part of the initial conditions. On the other hand, it could be that certain directions of space simply rolled themselves up at some point — spontaneous compactification — at a time somewhat after the big-bang.

A simple example of a Kaluza-Klein cosmology is provided by a generalization of the Kasner [15,16] solution. Suppose that the metric on large scales is given by

$$ds^2 = -dt^2 + \sum_{i=1}^{n-1} a_i^2(t) dx^i{}^2. \quad (1)$$

$x^i (i = 1, 2, 3)$ should be regarded as coordinates on R^3 representing the spatial part of the universe. $x^i (i = 4,$

$\dots, N-1)$ should be regarded as coordinates on T^{n-4} , the $(n-4)$ -dimensional torus giving an isometry group of $U(1)^{n-4}$. The vacuum Einstein equations then give

$$a_i(t) = (t/t_0)^{p_i}, \quad \sum p_i = 1, \quad \sum p_i^2 = 1. \quad (2)$$

It follows from this that at least one direction must be contracting unless the spacetime is flat. Presumably, this contraction will stop once the radius is of order the Planck scale since it would seem inevitable that quantum effects would become important. One can try to make a more realistic model by demanding that $p_1 = p_2 = p_3 \equiv p_A$ and $p_4 = \dots = p_{n-4} \equiv p_B$ in which case

$$3p_A + Dp_B = 1, \quad 3p_A^2 + Dp_B^2 = 1, \quad (3)$$

where now $D = n - 4$. Solving for p_A and p_B gives

$$p_A = \frac{1 \pm (\frac{1}{3}D^2 + \frac{2}{3}D)^{1/2}}{3 + D},$$

$$p_B = \frac{D \pm 3(\frac{1}{3}D^2 + \frac{2}{3}D)^{1/2}}{3 + D}. \quad (4)$$

Thus either $p_A > 0, p_B < 0$ or $p_A < 0, p_B > 0$. We will concentrate on the first case. We note the following three features.

First, the compact directions do not have constant proper spatial extent. Second, the point $t = 0$ is a spacelike singularity (rather like the big-bang) and finally, the Kaluza-Klein ansatz has been put in as an initial condition. A consequence of this is that monopoles appear as initial data since topological defects may or may not be ruled out a priori.

For the GUT monopole in the standard cosmology there is a temperature above which there are no monopoles, so the number of monopoles produced is independent of initial configurations of the Higgs field. For the Kasner cosmology discussed above the number density of monopoles depends on the initial orientation of compactification, and cannot be calculated. Even knowledge of the initial conditions (hence the initial monopole number density) is inadequate. The relevant parameter is the monopole-to-entropy ratio. The Kasner solution describes a "vacuum" solution, and if it is to be relevant it is necessary to create entropy either through particle creation in the anisotropic gravitational background [17-19] or through viscous damping

[20,21]. In either case the entropy produced depends on the details of the cosmology and the microphysics. The monopole-to-entropy ratio cannot be calculated in this class of cosmologies; neither the monopole density nor the entropy density is calculable.

A second example of a Kaluza–Klein cosmology is one in which the “spatial” directions are like those in a Robertson–Walker spacetime, and the “internal” directions are some space which is itself Einstein, with positive curvature. (If it does not have positive curvature, then it will fail to have any Killing vectors and not generate a satisfactory low-energy theory [22].) Such models have been considered previously by Freund [23], Sahdev [24], and Kolb, Lindley and Seckel [25]. The metric is taken to be

$$ds^2 = -dt^2 + a^2(t)g_{mn}dx^m dx^n + b^2(t)g_{\mu\nu}dx^\mu dx^\nu, \quad (5)$$

where $a(t)$ is the scale factor for the three observed spatial dimensions, $b(t)$ is the scale factor for the extra dimensions, $\tilde{g}_{mn}(x)$ is the metric for a maximally symmetric three-space, and $\tilde{g}_{\mu\nu}(y)$ is a metric on the compact internal space with positive scalar curvature. At early times the stress-energy tensor is assumed to have a $(4+D)$ -dimensional perfect fluid form with an equation of state for radiation.

A diagram describing the evolution of the scale factors for the “anisotropic” FRW case is shown in fig. 1. The scale factor associated with the extra dimensions reaches a maximum value, b_M , then decreases toward

a singularity ^{‡1}. After a time t_4 defined by $bT = 1$ (T is the $(3+D)$ -dimensional temperature) the universe is effectively four dimensional. One must assume that at some time $t_{\text{FRW}} \geq t_4$, the extra dimensions stabilize at some fixed radius. This presumably occurs due to quantum-mechanical effects. It will not occur if the effective energy–momentum tensor is that of a fluid.

The “anisotropic” FRW class of cosmologies has the feature that the effective three-dimensional entropy is not constant [24–28]. Even with the assumption that prior to t_4 the $n = (3+D)$ -dimensional expansion is isentropic, i.e.

$$a^3 b^D T^{D+3} = \text{constant} \quad (t < t_4), \quad (6)$$

the temperature may increase in the evolution, since the mean volume may decrease as b decreases. The increase in temperature (hence in entropy density) is presumably cut off at time $t = t_4$, when $bT < 1$. At this point there is insufficient thermal energy to excite the modes of the extra dimensions, the $(D+3)$ -dimensional perfect fluid term for T_{MN} is not relevant, and one may ignore the extra dimensions and consider four-dimensional cosmologies. At $t = t_4$ the excitations of the extra dimensions are realized as four-dimensional massive particles, which presumably decay ^{‡2}, thereby increasing the effective three-dimensional entropy.

^{‡1} This of course was guaranteed by the choice of a closed space, $k_D = +1$.

^{‡2} We assume here that all the excitations can decay. The possibility of stable excitations was considered in ref. [29].

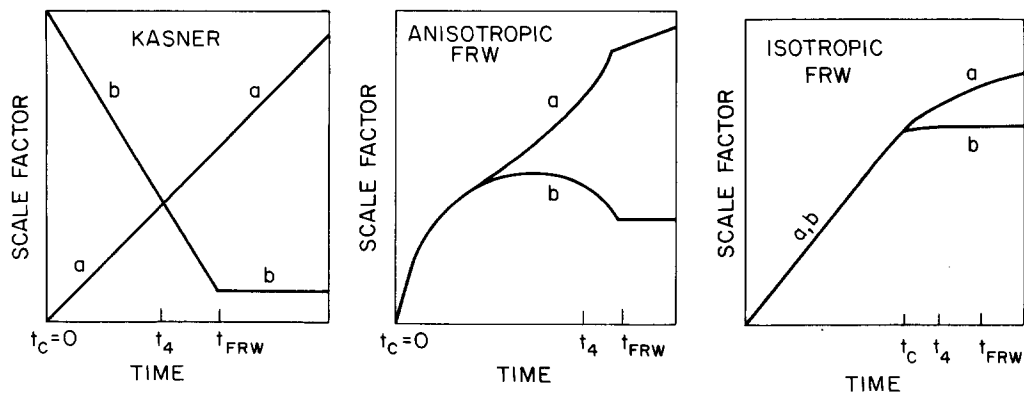


Fig. 1. Time evolution of scale factors for the three observed spatial dimensions (a) and for extra dimensions (b) in three Kaluza–Klein cosmologies.

Detailed calculations give an entropy, S_3 , in the horizon three-volume at $t = t_4$ of [25]

$$\ln S_3 \approx \frac{-2(D+3)[3/D(D+2)]^{1/2}}{1 - 2[3/D(D+2)]^{1/2}} \times \ln(R_{KK}/R_{P1}) \quad (D > 1) \\ \sim \ln(R_{KK}/R_{P1}) \quad (D \gg 3), \quad (7)$$

where R_{KK} is the radius of the compact extra dimensions. Although the entropy in the horizon three-volume increases while b decreases, the total entropy in a co-moving N -volume remains *constant* until $t = t_4$ (eq. (6)). Therefore at $t = t_4$, the monopole-to-entropy ratio must simply reflect its initial value since the number of monopoles in the co-moving volume is also conserved. In the absence of entropy production after $t = t_4$, the monopole-to-photon ratio today would simply reflect the initial value. The principal reason a monopole-to-entropy ratio cannot be predicted in this model is that the defects are present at $t = 0$, and hence they depend on the initial data.

The final cosmological model we consider is one in which the compactification occurs for dynamical reasons during the expansion. In this model, we assume the universe expands isotropically in $n + D + 3$ spatial dimensions from $t = 0$, and at $t = t_c$ there is a fluctuation in the geometry which causes D dimensions to compactify. In this case one may regard compactification as a phase transition and use the horizon distance as the order parameter. This would result in an average production of one monopole per horizon [3–5]. In an isotropic N -dimensional, radiation-dominated expansion, the horizon distance at compactification is ^{‡3}

$$d_H(t_c) = t_c \frac{n+1}{2} \left(\frac{n(n-1)\Gamma(n/2)\Gamma(D/2+1)}{32\pi^{2+n+D/2}\Gamma(n+1)} \right)^{1/2} \times \frac{M_{P1} M_{KK}^{D/2}}{T_c^{n+1/2}}, \quad (8)$$

where t_c is the time from the initial singularity, and we have assumed an isotropic expansion in $n = 3 + D$ spatial dimensions with a radiation dominated equation of state $\rho_n = 2\pi^{n/2} T^{n+1}/\Gamma(n/2) = n p_n$. We assume

^{‡3} For simplicity we assume the extra dimensions form a D -sphere with volume $v_D = R_{KK}^D \pi^{D/2}/\Gamma(D/2+1)$, where $R_{KK} \equiv M_{KK}^{-1}$ is the radius of the D -sphere.

that compactification occurs at $T_c = M_{KK}$, so that

$$d_H(t_c) = M_{P1}/M_{KK}^2, \quad (9)$$

which results in a monopole density of

$$n_M(T_c) = d_H^{-3}(T_c) \approx M_{KK}^6/M_{P1}^3. \quad (10)$$

If we assume that compactification is instantaneous, the photon density at compactification ($T = R_{KK}^{-1}$) would be

$$n_\gamma = V_D \rho_n = R_{KK}^D T^{n+1} = M_{KK}^3 \quad (R_{KK} = T). \quad (11)$$

Therefore the monopole-to-photon ratio in this model would be given by

$$n_M/n_\gamma = (M_{KK}/M_{P1})^3, \quad (12)$$

which is the same as the prediction of GUT monopoles if one takes $M_{KK} = M_{GUT}$. The Kaluza–Klein scale is expected to be close to the Planck scale so an initial monopole-to-photon ratio of $O(1)$ is predicted ^{‡4}.

For GUT monopoles, $M\bar{M}$ annihilations would reduce an initial n_M/n_γ of $O(1)$ to a final n_M/n_γ of $O(10^{-10})$ [3]. However, for the Kaluza–Klein monopoles, the $M\bar{M}$ system does not have the quantum numbers of the vacuum, and hence M and $\bar{M}$ cannot classically annihilate [7]. The fact that the $M\bar{M}$ system has non-trivial topology also implies that $M\bar{M}$ cannot classically be created in thermal processes. Of course in the final model we have invoked fluctuations of the geometry to achieve compactification; it is likely that similar fluctuations would allow $M\bar{M}$ pairs to be created or destroyed. In fact, such fluctuations are necessary to create the $M\bar{M}$ pairs in the compactification transition.

In this paper we have attempted to estimate the density expected for Kaluza–Klein monopoles based upon some simple cosmological models in more than four dimensions. The conclusion is that in some cosmologies the density depends upon initial data, hence unknown. In the final model the predicted monopole-to-photon ratio was of order unity. In either case it seems necessary to dilute the monopole density. The same dilution methods used for GUT monopoles may be used for Kaluza–Klein monopoles ^{‡5}. Inflationary

^{‡4} However there are compactification models with $M_{KK} = O(10^2 \text{ GeV})$ [30] in which case an initial $n_M/n_\gamma = 10^{-21}$ is expected.

^{‡5} For a review of monopole dilution mechanisms, see e.g. ref. [23].

dilution may be even more effective for Kaluza—Klein monopoles, because thermal production of $M\bar{M}$ pairs is (classically) forbidden.

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