

Worldbrane actions for string solitons

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The space-time dynamics of recently discovered fivebrane solitons in heterotic and type II string theories are described in terms of supersymmetric low-energy field theories which live on the $(5+1)$ -dimensional soliton worldbranes. Several surprising features emerge, including: the (hitherto unsuspected) existence of both chiral and non-chiral type II five-branes; spin-one gauge as well as anti-self-dual tensor zero modes; extended supersymmetry on the worldbrane. Non-renormalization theorems for supersymmetric worldbrane actions are used to argue that gauge fivebrane (and instanton) solutions to the leading-order low-energy string equation of motion correspond to exact supersymmetric solutions of the full string equation of motion.

1. Introduction

The study of solitons and instantons in string theory promises to yield interesting information about the non-perturbative, semi-classical structure of string theory. At least three field theories are involved in the study of these solutions; the two-dimensional sigma model on the string worldsheet, the six-dimensional (for the case of fivebranes) field theory on the soliton worldbrane which governs the low-energy dynamics of the worldbrane, and the string field theory itself (often studied in its low-energy limit). These field theories and their symmetries have fascinating interconnections.

In previous papers, ref. [1] and [2], fivebrane string solitons were studied mainly from the low-energy string field theory and worldsheet points of view respectively. In the present paper, we study the worldbrane field theories which govern the space-time dynamics of the solitons at large distances. These field theories will be

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given by six-dimensional (extended) supersymmetric sigma models whose target spaces are the moduli spaces of soliton solutions. This approach is somewhat analogous to the study of string compactification utilizing the structure of four-dimensional low-energy supersymmetric actions. This low-energy point of view allows one to make general and powerful arguments for the existence of solutions to the full string equations of motion without detailed knowledge of the precise form of the solutions. The subject of worldbrane actions is also interesting in its own right [3], and we will find some new results in this subject. “Static gauge” will be used throughout, and we will not tackle the difficult and interesting problem of covariant fivebrane actions.

The outline of this paper is as follows. In sect. 2 we review the solutions of refs. [1,2] and then in sect. 3 we discuss the zero modes of these solutions, their low-energy worldbrane actions, and the representations of six-dimensional supersymmetry which can arise in the various worldbrane actions. In sect. 4 we discuss the worldbrane action for the “gauge fivebrane” found in ref. [1] as a solution of the leading-order low-energy heterotic string equation of motion. We show that there is in fact a degenerate family of exact solutions of string theory labelled by (among other moduli) the scale size ρ of the Yang–Mills instanton residing at the fivebrane core. This result may come as a surprise: one does not expect a theory with dimensionful couplings, such as string theory, to have degenerate solutions labelled by a dimensionful parameter. We will see that this surprise can be understood as a consequence of the low-energy supersymmetry of the worldbrane action.

The conformal field theory underlying the symmetric solution of ref. [2] yields a soliton for type II string theory as well. General arguments following from the representation theory of six-dimensional $N = 2$ supersymmetry imply that type II fivebranes must have unusual bosonic zero modes corresponding to a vector gauge field in the case of IIB and to an anti-self-dual tensor in the case of IIA. These fivebranes seem to lie outside the usual classification of consistent p -brane theories*. In sect. 5 we discuss the corresponding type II fivebranes from the low-energy string field theory point of view and explicitly construct the zero modes. We cannot refrain, in sect. 6, from speculating on the implications of these results for the string/fivebrane duality conjectured in ref. [1]. In appendix A we explain a peculiar chirality flip in gauge fixing to static gauge which clarifies the discussion of the type II solutions.

2. Preliminaries

We begin with a summary of the three solutions of refs. [1,2,4,5]. The low-energy action for the bosonic degrees of freedom of the heterotic string is given in

* A nice review of p -branes can be found in ref. [3].

sigma-model variables by

$$S_{10} = \frac{1}{\alpha'^4} \int d^{10}x \sqrt{-g} e^{-2\phi} \left(R + 4(\nabla\phi)^2 - \frac{1}{3}H^2 - \frac{1}{30}\alpha' \text{Tr } F^2 \right) \quad (2.1)$$

plus higher-order α' corrections. Our conventions are those of ref. [2]. The full action is invariant under supersymmetry transformations, under which the fermions transform to leading order in α' as

$$\begin{aligned} \delta\psi_M &= \nabla_M \epsilon - \frac{1}{4}H_{MAB}\Gamma^{AB}\epsilon, \\ \delta\lambda &= \Gamma^M \nabla_M \phi \epsilon - \frac{1}{6}H_{ABC}\Gamma^{ABC}\epsilon, \\ \delta\chi &= F_{MN}\Gamma^{MN}\epsilon, \end{aligned} \quad (2.2)$$

where ψ_M , λ and χ are the gravitino, dilatino and gaugino, respectively. A “vacuum” fivebrane soliton solution is a solution with $O(5, 1)$ symmetry which is asymptotically flat in the 4 transverse dimensions. A supersymmetric solution is one for which in addition the supersymmetry transformations (2.2) vanish for some choice of ϵ .

The “gauge solution” [1] begins with a one-instanton configuration of scale size ρ obeying

$$F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu}{}^{\rho\sigma}F_{\rho\sigma}, \quad (2.3)$$

for $F_{\mu\nu}$ lying in an $SU(2)$ subgroup of $E_8 \times E_8$ or $SO(32)$ (we use $\mu, \nu = 6, \dots, 9$ to denote indices tangent to the space transverse to the fivebrane, and $a, b = 0, \dots, 5$ the orthogonal indices.) Let

$$\begin{aligned} A_\mu &= \frac{-2\Sigma_{\mu\nu}x^\nu}{(x^2 + \rho^2)}, \\ e^{2\phi} &= e^{2\phi_0} + 8\alpha' \frac{(x^2 + 2\rho^2)}{(x^2 + \rho^2)^2}, \\ H_{\mu\nu\lambda} &= -\epsilon_{\mu\nu\lambda}{}^\rho \nabla_\rho \phi, \\ g_{ab} &= \eta_{ab}, \\ g_{\mu\nu} &= e^{2\phi} \delta_{\mu\nu}, \end{aligned} \quad (2.4)$$

where $\Sigma_{\mu\nu}$ is antisymmetric and self-dual. This configuration is a solution of the

equations of motion and is annihilated by supersymmetry transformations generated by spinors ϵ_+ obeying

$$\begin{aligned}\epsilon_{\mu\nu\lambda\rho}\Gamma^{\mu\nu\lambda\rho}\epsilon_+ &= 24\epsilon_+ \\ \epsilon_{abcdef}\Gamma^{abcdef}\epsilon_+ &= 720\epsilon_+.\end{aligned}\tag{2.5}$$

it therefore represents a supersymmetric fivebrane soliton. A four-dimensional cross-section of this soliton is a standard Yang–Mills instanton, dressed up with a non-trivial dilaton and axion and with a non-flat metric.

There are two “charges” one can associate with this solution. These are the instanton winding number

$$\nu = \frac{1}{480\pi^2} \int \text{Tr } F \wedge F, \tag{2.6}$$

where the integral is over a four-dimensional cross section, and the axion charge

$$Q = -\frac{1}{2\pi^2} \int H, \tag{2.7}$$

where the integral is over an asymptotic S^3 surrounding the fivebrane. These charges are both quantized [6], with the minimal allowed values given by $\nu = 1$ and $Q = \alpha'$. For the solution (2.4) these charges are

$$\nu = 1, \quad Q = 8\alpha'. \tag{2.8}$$

The “neutral solution” [2,4] is given by the configuration

$$\begin{aligned}A_\mu &= 0, \\ e^{2\phi} &= e^{2\phi_0} + \frac{n\alpha'}{x^2}, \\ H_{\mu\nu\lambda} &= -\epsilon_{\mu\nu\lambda}{}^\rho \nabla_\rho \phi, \\ g_{ab} &= \eta_{ab}, \\ g_{\mu\nu} &= e^{2\phi} \delta_{\mu\nu},\end{aligned}\tag{2.9}$$

where n is an integer. The charges take the values

$$\nu = 0, \quad Q = n\alpha'. \tag{2.10}$$

The metric in (2.9) is everywhere smooth (despite a coordinate singularity at $x = 0$), as can be easily seen by transforming to coordinates $t = \ln x^2$. A four-di-

mensional cross section of this fivebrane reveals a semi-wormhole throat at the core, whose radius asymptotically approaches $\sqrt{Q}$. The space-time topology of this soliton is $S^3 \times \mathbb{R} \times M^6$, where M^6 is six-dimensional Minkowski space.

The “symmetric solution” [2] is given by

$$\begin{aligned}
 A_\mu &= \frac{-2 \sum_{\mu\nu} x^\nu}{(x^2 + n\alpha' e^{-2\phi_0})}, \\
 e^{2\phi} &= e^{2\phi_0} + \frac{n\alpha'}{x^2}, \\
 H_{\mu\nu\lambda} &= -\epsilon_{\mu\nu\lambda}{}^\rho \nabla_\rho \phi, \\
 g_{ab} &= \eta_{ab}, \\
 g_{\mu\nu} &= e^{2\phi} \delta_{\mu\nu}.
 \end{aligned} \tag{2.11}$$

This solution has

$$\nu = 1, \quad A = n\alpha'. \tag{2.12}$$

It, too, has a semi-wormhole at the fivebrane core. As discussed at length in ref. [2] this solution has the spin connection with H -torsion embedded in the gauge group in just such a way as to lead to (4,4) supersymmetry on the string worldsheet. This implies that the underlying conformal field theory provides a solution for type II as well as heterotic strings, a fact which shall be exploited in sect. 5.

While the geometry described by the neutral and symmetric solutions is everywhere smooth, the reader may nevertheless be concerned that the existence of a second asymptotic region and the unboundedness of ϕ at $x = 0$ might preclude a physical interpretation of (2.9) or (2.11) as a fivebrane with sensible dynamics. We shall see that this is not the case. They both have a finite mass per unit five-volume and the zero modes corresponding to small excitations are normalizable. Physics outside the fivebrane core is insensitive to boundary conditions at $x = 0$ because it is infinitely far away.

However the neutral and symmetric solutions do differ from the gauge solutions (2.4) in one crucial respect. A pair of oppositely charged parallel fivebranes of the gauge type (2.4) is topologically trivial and can be continuously deformed to the vacuum. This means that they can be pair produced, for example, in the early universe, and their inclusion in string theory is obligatory. In contrast, the space-time topology of a pair of solutions of the type (2.9) or (2.11) is never equivalent to the Minkowski vacuum topology. It is therefore not obvious if they can be pair produced or if their inclusion in string theory is obligatory.

In this connection, we note that the geometry of the solutions (2.9) and (2.11) bears some resemblance to the extreme Reissner–Nordström geometry (in which

the horizon is an infinite distance along a static spatial slice from any exterior point), and an even closer resemblance to the stringy four-dimensional black holes discussed in refs. [7,8]. Motivated by this analogy it was shown in ref. [9] that (2.9) and (2.11) are in fact, in a sense made precise in ref. [9], higher-dimensional extended black holes. It is thus quite plausible that they might be produced classically via gravitational collapse. In this case their inclusion in string theory is obligatory.

3. Zero modes, worldbrane actions and $D = 6$ supersymmetry

The solutions described above are not unique but belong to multi-parameter families of solutions labelled by a set of collective coordinates arising from zero modes. In this section we will describe these zero modes and collective coordinates and the low-energy action which governs their dynamics.

An efficient way of describing a family of degenerate solutions is in terms of an effective action, S_{eff} . S_{eff} is determined by the property that its stationary points describe the classical motion of a fivebrane on scales large compared to both $\sqrt{\alpha'}$ and the thickness ρ of the fivebrane. S_{eff} has the general form

$$S_{\text{eff}} = \frac{1}{\alpha'^4} \int d^{10}x \sqrt{-g} e^{-2\phi} (R + \dots) + \int d^6\sigma e^{-2\phi} \mathcal{L}_{\text{wb}}(g_{MN}(X), \dots; X^M(\sigma), \dots), \quad (3.1)$$

where σ is a coordinate on the worldbrane. The first term depends only on space-time fields such as the metric, while the worldbrane lagrangian $\mathcal{L}_{\text{wb}}$ depends on both space-time fields and worldbrane fields such as the location $X^M(\sigma)$ of the point σ on the worldbrane. Varying S_{eff} with respect to the space-time fields leads to the usual massless field equations for the heterotic string supplemented by a source term with support on the worldbrane. Varying with respect to the worldbrane fields leads to the classical equation of motion for a fivebrane in the background given by the massless fields. The dilaton prefactor in front of the worldbrane action has been extracted in order to ensure that shifting the dilaton field by a constant is a symmetry of the full space-time plus worldbrane action.

The worldbrane action will contain one massless field for every zero mode of the fivebrane soliton solution. In principle the complete worldbrane action can be derived by a dimensional reduction procedure involving an expansion in small fluctuations around the static solution. Let Φ_0 denote collectively the massless field configuration describing the static fivebrane solution

$$\left. \frac{\delta S_{10}}{\delta \Phi} \right|_{\Phi_0} = 0, \quad (3.2)$$

with S_{10} given by (2.1). Let $\phi_\alpha(x^\mu)$ be a zero mode obeying

$$\square_0 \phi_\alpha = 0, \quad (3.3)$$

where

$$e^{-2\phi} \square_0 \equiv \left. \frac{\delta^2 S_{10}}{\delta \Phi^2} \right|_{\Phi_0} \quad (3.4)$$

is the kinetic operator in the Φ_0 background. Consider perturbing the solution Φ_0 by the zero mode multiplied by a collective coordinate field $\lambda^\alpha(\sigma^a)$ which varies slowly along the worldbrane. It is convenient here to use “static” or “physical” gauge for which $X^a = \sigma^a$. The action S_{10} then becomes

$$S_{10}[\Phi_0 + \lambda^\alpha \phi_\alpha] = S_{10}[\Phi_0] + \frac{1}{2} \int d^{10}x \sqrt{-g} e^{-2\phi} \lambda^\alpha \phi_\alpha \square_0 \lambda^\beta \phi_\beta + O(\lambda^3). \quad (3.5)$$

Defining a metric $G_{\alpha\beta}$ on the space of zero modes by

$$G_{\alpha\beta} = \int d^4x \sqrt{g} e^{-2\phi} \phi_\alpha \phi_\beta, \quad (3.6)$$

one has to leading order

$$S_{10}[\Phi_0 + \lambda^\alpha \phi_\alpha] = S_{10}[\Phi_0] + \frac{1}{2} \int d^6\sigma G_{\alpha\beta} \lambda^\alpha \square_0 \lambda^\beta + \dots \quad (3.7)$$

The second term may then be identified as the leading-order worldbrane action in static gauge. There are two subtleties in this discussion. First, note that the worldsheet action (3.7) appears to be missing a factor of $e^{-2\phi}$ compared to (3.1). This is an illusion: one can extract a factor of $e^{-2\phi_0}$ from the metric $G_{\alpha\beta}$ in (3.6) and, to reproduce (3.1), one has only to imagine that ϕ_0 is not actually constant, but slowly varying on the scale of the transverse size of the fivebrane. The second subtlety concerns the metric on the space of bosonic zero modes. The zero modes describe physical motions of the system in those special directions in which there are no restoring forces (i.e. tangent to the moduli space). Because of the gauge invariances and associated constraints the zero modes are not simply derivatives of the static solutions with respect to the modular parameters and a little effort is required to identify them completely. On the other hand, the metric of (3.6) is the metric on moduli space which, as explained in ref. [10], governs the dynamics of slowly moving fivebranes and is therefore an object of considerable intrinsic physical interest. A calculation of its properties is underway and will be reported elsewhere [11].

In principle the complete non-linear worldbrane action may be in this manner determined by continuing the expansion in λ^α . In practice it is easier to deduce it from the many symmetries that it must respect. Worldbrane symmetries descend to the worldbrane from space-time symmetries. Space-time symmetries which are broken by the soliton solution are non-linearly realized on the worldbrane, as discussed in detail in ref. [12]. For example, 4 of the 10 translational symmetries and 8 of the 16 supersymmetries are broken by the soliton solution. The static gauge worldbrane action accordingly contains 4 Goldstone bosons (X^μ) as well as 8 goldstinos. Space-time symmetries which are unbroken (in our case the six-dimensional Poincaré group and half of the supersymmetries) on the other hand are linearly realized as worldbrane symmetries in static gauge [12]. In this regard static gauge simplifies the analysis since in this gauge the worldbrane action will be that of a standard supersymmetric sigma model with the supersymmetry descending from the unbroken space-time supersymmetries. In covariant gauge on the other hand the unbroken space-time symmetries becomes κ symmetries.

In order to understand the possible forms that the six-dimensional worldbrane supersymmetry can take it is useful to recall a few facts about six-dimensional supersymmetry [13,14]. Chiral (p, q) representations of supersymmetry exist in 2, 6 and 10 dimensions. The minimal $N = 1$ $((0, 1)$ or $(1, 0))$ representation of ten-dimensional supersymmetry is generated by a Majorana–Weyl spinor with 16 real components. There are two $N = 2$ theories, IIA $(1, 1)$ and IIB $(0, 2)$.

The minimal $(0, 1)$ representation of six-dimensional supersymmetry is generated by a single Weyl spinor which has 8 real components. The supersymmetry algebra has an $SU(2)$ automorphism group which allows one to re-package the 8 real components in terms of a spinor field ψ_α which is a Weyl spinor of the covering group of the six-dimensional Lorentz group $(SO(5, 1) \cong SI(2; \mathbb{H}))$ with $\mathbb{H}$ the quaternions) and also a spinor of $SU(2)$ with $SU(2)$ spinor index α . In order for ψ_α to have only 8 real independent components it must obey a “symplectic Majorana condition

$$\psi_\alpha = \epsilon_{\alpha\beta} \psi_\beta^C, \quad (3.8)$$

with ψ^C the $SO(5, 1)$ charge conjugate spinor.

The supersymmetry multiplets consist of the gravity multiplet, and three different matter multiplets, Φ , A and B . Each of these matter multiplets contains a doublet of Weyl fermions, but they differ in their boson content. Φ contains four scalars, A contains a single vector field and B contains an anti-self-dual tensor and a single scalar.

There are again two $N = 2$ theories in $D = 6$, $(1, 1)$ and $(0, 2)$. The $(1, 1)$ theory has a single matter multiplet which consists of the Φ plus A multiplets of the $N = 1$ theory. The $(0, 2)$ theory has a matter multiplet which is the sum of the Φ and B multiplets. In order to see which of these representations we will find in the

fivebrane action we can decompose the fields under a $SO(5, 1) \otimes SO(4)$ subgroup of $SO(9, 1)$. Under this subgroup the Majorana–Weyl supersymmetry parameter $\epsilon_{\pm}$ of the $D = 10$ $(1, 0)$ or $(0, 1)$ theory (with $\Gamma_{11}\epsilon_{\pm} = \pm\epsilon_{\pm}$) decomposes as

$$16_{\pm} \rightarrow (4_+, 2_{\pm}) \oplus (4_-, 2_{\mp}). \quad (3.9)$$

The spinor representations of $SO(5, 1)$, $4_{\pm}$, like the spinor representations of $SU(2)$, are pseudoreal. The Majorana condition on $\epsilon_{\pm}$ imposes the generalized symplectic Majorana condition (3.8) on the spinors $(4_{\pm}, 2_{\pm})$ which implies that they have only 8 real independent components.

The fivebrane solutions to heterotic string theory (2.4), (2.9) and (2.11) all preserve half of the $N = 1$, $D = 10$ supersymmetry. Their low-energy fivebrane action will thus have a linearly realized $(0, 1)$ $D = 6$ supersymmetry with the supercharge transforming as $(4_+, 2_+)$. The symmetric solution (2.11) has $(4, 4)$ worldsheet supersymmetry and thus provides a solution for type II string theory as well. Depending on whether we choose IIA $((1, 1))$ or IIB $((0, 2))$ we will be led to a fivebrane action with either $(0, 2)$ or $(1, 1)$ supersymmetry in $D = 6$. The surprising fact that the non-chiral $D = 10$ theory leads to a chiral $D = 6$ theory and vice versa is discussed in sect. 5 and appendix A.

We now discuss the various zero modes and worldbrane actions starting with the simplest case: the neutral solution (2.9) as a fivebrane in heterotic string theory. We shall refer to this solution as the “neutral fivebrane”. This neutral fivebrane has only four bosonic zero modes corresponding to the four translation symmetries which are broken by the static solution. In static gauge

$$X^a = \sigma^a \quad a = 0, 1, \dots, 5, \quad (3.10)$$

these lead to the worldbrane action

$$(N/\alpha'^3) \int d^6\sigma \, e^{-2\phi} \eta^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}, \quad (3.11)$$

where $\mu, \nu = 6, \dots, 9$ and N is a normalization factor. This is a gauge fixed-form of the Polyakov or Nambu action. In addition, of the 16 supertranslation symmetries of ten-dimensional Minkowski space, exactly half are broken by the fivebrane solution. These give eight fermionic zero modes

$$Z_q = \delta_{\epsilon_q} \Psi \quad (3.12)$$

where ϵ_q , $q = 1, \dots, 8$, are the eight spinors generating the broken supersymmetries, Ψ denotes all the space-time Fermi fields. These zero modes lead to eight massless worldbrane fermions, which may be assembled into a single six-dimensional symplectic Majorana–Weyl doublet λ .

Because of the uncertain effects of the “singularity” at $x = 0$, it is important to check that the zero modes are normalizable. This is indeed the case for all the zero modes discussed in this paper. In checking this, it is important to include a factor of $e^{-2\phi}$ in the zero mode inner product which is implied by (2.1). The normalizability of the zero modes implies that the excitations are localized (with respect to the relevant measure) near the juncture of the semi-wormhole with the $\mathbb{R}^4$, rather than near $x = 0$.

It is easily seen that worldbrane field theory indeed provides a representation of the space-time supersymmetry algebra [12,15]. The eight broken supersymmetries are non-linearly realized, with λ transforming as a goldstino. The eight unbroken supersymmetries, on the other hand, are linearly realized in static gauge as global, $N = 1$ six-dimensional supersymmetry. This implies, as we have found explicitly, that the number of fermion zero modes must be twice the number of bosonic zero modes.

The action described so far is not invariant under ten-dimensional Lorentz transformations because of the choice of static gauge. The problem of finding a fully covariant action for the neutral fivebrane which reduces to this gauge-fixed action has been solved in [15–17]. The covariant action is a six-dimensional generalization of the Green–Schwarz superstring action containing 10 commuting fields X^M and 16 anticommuting fields Θ which transform as a single Majorana–Weyl spinor under ten-dimensional Lorentz transformations.

Next let us consider the fivebrane arising from the gauge solution (2.4) in heterotic string theory. This “gauge fivebrane” was discussed in refs. [1] and [18]. The number of fermion zero modes is determined by the Atiyah–Singer index theorem, for either $E_8 \times E_8$ or $SO(32)$, to be 240. This number includes the 8 supertranslation zero modes. There are also 120 bosonic zero modes which parametrize the moduli space of E_8 or $SO(32)$ instantons on $\mathbb{R}^4$. In static gauge, the worldbrane action is the $D = 6$, $N = 1$ supersymmetric sigma model with the instanton moduli space $\mathcal{M}$ as a target manifold. The bosonic and fermionic zero modes combine into 30 Φ multiplets under the $N = 1$ supersymmetry.

The symmetric solution (2.11) contains the same 240 fermionic and 120 bosonic zero modes. We note that one of the 120 bosonic zero modes is a dilational zero mode for the gauge fields which breaks the $(4, 4)$ down to $(0, 4)$ symmetry, and that inclusion of this mode is essential for worldbrane supersymmetry. However, as in the case of string compactification, there may be extra massless modes at the symmetric points of the moduli space arising from enhanced worldsheet symmetries. It appears that this is the case for the symmetric solution (2.11). This question is currently under investigation [19].

4. A non-renormalization argument

It was argued in ref. [2] that the symmetric solution (2.11) is an exact solution to heterotic string theory without any higher-order corrections in α' . On the other hand, explicit analysis [2] of the supersymmetry transformation laws at subleading order in α' [20] shows that the functional form (2.4) of the leading-order gauge solution is not exact, but must be corrected order by order in α' . We would like to know whether these higher-order corrections can destabilize the solution or whether, as in the case of (2, 2) Calabi–Yau compactifications, they simply re-arrange the massive fields but leave one with a solution to string theory.

This question can be answered by an analysis of the six-dimensional worldbrane field theory governing the gauge fivebrane. As discussed in sect. 3, this theory is comprised of scalar and spinor fields in the Φ multiplet of (0, 1) six-dimensional supersymmetry. The most general such theory has been constructed in ref. [21]. The scalar fields must form a non-linear sigma model with a hyperkähler target space. In our case there are 120 scalar fields with the E_8 or $SO(32)$ instanton moduli space $\mathcal{M}$ as a target space. $\mathcal{M}$ indeed is known to have a hyperkähler structure induced from the hyperkähler structure on $\mathbb{R}^4$. The leading-order world-brane action is given by

$$S_{\text{wb}} = (N/\alpha'^3) \int d^6\sigma \, \eta^{ab} (G_{\alpha\beta}(Z) \, \partial_a Z^\alpha(\sigma) \, \partial_b Z^\beta(\sigma) + \text{fermionic terms}), \quad (4.1)$$

where Z is a coordinate and G the hyperkähler metric on $\mathcal{M}$ and N is a normalization factor.

Among the fields Z in (4.1) is the field ρ corresponding to dilations of the fivebrane core. The fact that, to leading order in α' , there is a degenerate solution for every value of ρ is reflected in the absence of a potential for ρ in (4.1). Higher-dimension corrections to (4.1) arise from two sources: higher-dimension corrections to the space-time action (2.1) and higher-order terms from the dimensional reduction in (3.7). These corrections can be computed as a double series in α' and $1/\rho$. Generically one might expect that ρ is not an exact flat direction and that at higher orders a potential for ρ will appear of the form

$$V_{\text{wb}} = \int d^6\sigma \, e^{-2\phi} v(\rho), \quad (4.2)$$

where the dilaton pre-factor has been determined by the scaling arguments following (3.1). Such a term would imply that the mass per unit five-volume of the fivebrane depends on its thickness. (4.2) would further imply that a fivebrane acts as a source of scalar dilaton radiation, since the dilaton equation of motion

acquires the extra term

$$\frac{\delta S_{10}}{\delta \phi(x)} = 2 \int d^6 \sigma \delta^{10}(x - X(\sigma)) e^{-2\phi} v(\rho). \quad (4.3)$$

A fivebrane of thickness ρ would then decrease its mass by the emission of dilaton radiation until its size reached the minimum of the potential $v(\rho)$. On dimensional grounds this minimum would be expected to be at ρ^2 equal to 0, infinity or order α' .

A non-vanishing potential for the field ρ would imply that a static solution does not exist for every value of ρ . Thus the question of whether or not the leading-order solutions of ref. [1] correspond to an exact solution for every value of ρ is equivalent, in the language of effective low-energy field theory, to the question of whether or not a potential is generated for ρ .

In fact such a potential can not be generated for the following reason. At string tree-level, the total low-energy effective action (3.1) must be supersymmetric, though it may lead to spontaneous supersymmetry breaking. In particular when the space-time fields take their constant values corresponding to flat ten-dimensional Minkowski space, the full worldbrane action (including all corrections in α' , both perturbative and non-perturbative) must exhibit $D = 6, (0, 1)$ supersymmetry. This means that the term (4.3) cannot appear by itself, but can at best appear as part of a superpotential accompanied by superpartners. However in ref. [21] it is shown that the $D = 6, (0, 1)$ non-linear sigma model does not admit any superpotential at all!

In conclusion, a non-trivial potential for the field ρ is completely ruled out by low-energy supersymmetry. Therefore the leading-order fivebrane solution of equation (2.4) provides the basis for an exact solution of string theory in the sense that one can construct an exact solution order by order in α' . A nearly identical argument with Z replaced by coordinates on the multi-instanton moduli space implies that higher-charge fivebranes also provide a basis for the construction of exact solutions.

An important assumption in the preceding is that, as argued in refs. [22,23], supersymmetry is unbroken for string theory in ten-dimensional flat Minkowski space (without fivebranes). To the extent that this assumption remains valid at the quantum level, we expect our arguments to be unaffected by quantum corrections.

The informed reader may have noticed the similarity between our argument and those given in ref. [24] for the existence of solutions based on Calabi–Yau sigma models and their generalizations with $(0, 2)$ worldsheet supersymmetry. These solutions were first found as solutions to the low-energy heterotic field equations, and are part of a one-parameter family labelled by the radius R of the six-dimensional space. R becomes a massless field in the low-energy four-dimensional effective field theory. The question of existence of an exact static solution is then

rephrased as the question of whether or not a space-time potential is generated for R . In the case of $(0, 2)$ compactifications, four-dimensional supersymmetry requires that any such potential be expressible in terms of a holomorphic function of $R + ia$, where a is an axion. Such functions are then ruled out *perturbatively* by the perturbatively valid Peccei–Quinn symmetry. Non-perturbatively, this symmetry is not in general valid. A potential for R can then be generated, and in some cases exact static solutions are known *not* to exist for every value of R . For $(2, 2)$ models, which lead to $N = 2$ space-time supersymmetry in the context of type II string compactification, the constraints are stronger and exact solutions do exist for every R . In this argument, the field R plays the role played by the field ρ in the present case, and four-dimensional space-time supersymmetry plays the role of six-dimensional worldbrane supersymmetry.

We further note that our results are in harmony with the observation that the fivebrane soliton saturates a Bogolmony bound relating the mass per unit five-volume to the axion charge Q [1,25]. One expects that, as in ref. [26], the Bogolmony bound should not be affected by higher-dimension corrections. This is certainly consistent with the absence of a potential for ρ .

5. Type II fivebranes

As mentioned above, and described in detail in ref. [2], the symmetric solution (2.11) gives rise to a $(4, 4)$ superconformal field theory and thus must provide a solution to type II string theory as well. From the representation theory of $N = 2$ supersymmetry in $D = 6$ we know that these solutions must have bosonic zero modes which are not $SO(5, 1)$ scalars. In this section we will see explicitly how these zero modes arise from the space-time field-equations. We will start with the “type IIB fivebrane” which arises as a soliton of the the chiral type IIB string theory.

5.1. TYPE IIB FIVEBRANE

The IIB theory involves a five-form field strength F which is self-dual. Because of this there is no simple covariant action for the IIB theory. The field content, equations of motion, and on-shell supersymmetry transformation laws are known and are sufficient to discuss the general properties of the solution. The bosonic fields that enter into the fermion transformation laws are, in addition to the metric and F , a three-form field strength G_{MNP} and a one-form P_M (which is constructed in terms of derivatives of a scalar field). The supersymmetry transformation laws of the fermion fields in the soliton background (which has $F = 0$) are given by

$$\begin{aligned}\delta\lambda &= i\Gamma^M P_M \epsilon^* - \frac{1}{24}i\Gamma^{MNP} G_{MNP} \epsilon \\ \delta\psi_M &= D_M \epsilon + \frac{1}{96}\left(\Gamma_M{}^{NPQ} G_{NPQ} - 9\Gamma^{NP} G_{MNP}\right)\epsilon^*\end{aligned}\tag{5.1}$$

in the notation of refs. [27,28] (with $\kappa = 1$ and $\Gamma_{11}\epsilon = -\epsilon$). The metric which appears in these equations is the “standard” general relativity metric and is related to the sigma model metric appearing in (2.1) by a factor of $e^{-\phi/2}$. For the fivebrane soliton, the non-zero components of G and P are

$$\begin{aligned} P_\mu &= \frac{1}{2} \nabla_\mu \phi, \\ G_{\mu\nu\rho} &= -2\epsilon_{\mu\nu\rho}{}^\lambda \nabla_\lambda \phi, \end{aligned} \quad (5.2)$$

where

$$\begin{aligned} e^{2\phi} &= e^{2\phi_0} + \frac{n\alpha'}{x^2}, \\ g_{\mu\nu} &= e^{3\phi/2} \delta_{\mu\nu}, \\ g_{ab} &= e^{-\phi/2} \eta_{ab}. \end{aligned} \quad (5.3)$$

For this field configuration, $\delta\psi$ and $\delta\lambda$ vanish for 16 choices of the spinor ϵ . The other 16 generate 16 fermion zero modes. These 16 fermion zero modes can be grouped into two six-dimensional symplectic Majorana–Weyl doublets of *opposite* chirality. This chirality flip can be easily seen by writing the complex Weyl spinor ϵ appearing in the supersymmetry transformation law (5.1) in terms of its real and imaginary parts $\epsilon = \epsilon_1 + i\epsilon_2$. For example the dilatino transformation in the soliton background then reads

$$\delta\lambda = \frac{1}{2}i \left[\left(\Gamma^\mu \nabla_\mu \phi + \frac{1}{6} \Gamma^{\mu\nu\rho} \epsilon_{\mu\nu\rho}{}^\lambda \nabla_\lambda \phi \right) \epsilon_1 - i \left(\Gamma^\mu \nabla_\mu \phi - \frac{1}{6} \Gamma^{\mu\nu\rho} \epsilon_{\mu\nu\rho}{}^\lambda \nabla_\lambda \phi \right) \epsilon_2 \right]. \quad (5.4)$$

Both ϵ_1 and ϵ_2 transform as $(4_-, 2_+) + (4_+, 2_-)$ and we see from eq. (5.4) that the unbroken supersymmetries are given by the 2_+ part of ϵ_1 and the 2_- part of ϵ_2 . It is perhaps surprising that the chiral IIB theory leads to a non-chiral worldbrane theory. The consistency of this with the fact that the chiral $D = 10$ IIB supersymmetry algebra must have a partially non-linear realization on the worldbrane is discussed in appendix A.

Worldbrane supersymmetry implies that in static gauge the number of Fermi fields must be twice the number of boson fields, so we need eight Bose zero modes. There are the usual four bosonic translation zero modes, but we need another four. The four extra zero modes are contained in the excitations of the self-dual rank-five antisymmetric tensor field

$$F = *F = dA. \quad (5.5)$$

where irrelevant terms involving G are omitted. This is a Ramond–Ramond field which does not appear in heterotic string theory. The zero modes turn out to be

pure gauge at zero worldbrane momentum, so for clarity we give the general expression at non-zero momentum. Let the momentum k be a self-dual constant null vector in two lorentzian dimensions tangent to the worldbrane, $\hat{*}$ the Hodge dual in the eight transverse dimensions and E a polarization vector orthogonal to k but tangent to the worldbrane. The zero modes are then given by

$$A_0 = e^{ik \cdot x} (E \wedge G + \hat{*} (E \wedge G)). \quad (5.6)$$

It is easy to check that $F_0 = dA_0$ is a self-dual five-form. Alternately, one may choose k to be anti-self-dual and insert a relative minus sign between the two terms in A_0 . These zero modes lead to a rank-two (or equivalent rank-four) field strength on the worldbrane describing four spin-one degrees of freedom. Together with the other zero modes, these fields comprise the $D = 6$ (1, 1) matter supermultiplet.

The type IIB fivebrane is a surprise for at least two reasons. First, the repeatedly discussed [16,18] “brane-scan” of classically consistent super p -branes excludes such an object. Apparently the assumptions leading to this brane-scan are too strong to apply to the present case. Secondly, to our knowledge, this is the first example of a solitonic p -brane with worldbrane spins greater than one-half. The difficulties in obtaining higher-spin zero modes has a severe obstacle for efforts to model our universe as a threebrane embedded in a higher-dimensional space. A construction similar to ours starting in eight dimensions would lead to a three-brane with abelian gauge fields. Unfortunately it seems unlikely that we would be able to obtain spin-two zero modes in this way. The spin-one zero modes are forced on us by supersymmetry. To obtain spin-two zero modes in this way we would need $N = 8$ supersymmetry in four dimensions. But this cannot arise from partial breaking of supersymmetry in ten or eleven dimensions.

The leading term in the low-energy effective action for the type IIB string theory has a global $SU(1, 1)$ symmetry. Naïvely, one might expect to obtain new fivebranes by continuous global rotations of the solution discussed herein. This is in fact not possible in string theory because the quantization condition on the axion charge $Q = -(1/2\pi^2)\int H$ (which lies in an $SU(1, 1)$ doublet) allows only a discrete subgroup of $SU(1, 1)$. It is possible that new, consistent fivebranes are obtained for these discrete rotations.

5.2. THE IIA FIVEBRANE

The non-chiral type IIA fivebranes has an equally intriguing structure. The IIA string theory [29] contains a two-form field strength G and a four-form field strength F as well as the three-form H . The bosonic part of the low-energy action

is (in sigma-model variables)

$$S = (1/\alpha'^4) \int d^{10}x \sqrt{-g} \left[e^{-2\phi} \left(R + 4(\nabla\phi)^2 - \frac{1}{3}H^2 \right) - \alpha' G^2 - \frac{1}{12} \alpha' F'^2 \right. \\ \left. - \frac{1}{288} \alpha' \epsilon^{M_1 \dots M_{10}} F_{M_1 M_2 M_3 M_4} F_{M_5 M_6 M_7 M_8} B_{M_9 M_{10}} \right]. \quad (5.7)$$

where $G = dA$, $H = dB$, $F = dC$ and $F' = dC + 2A \wedge H^*$. The supersymmetry transformation laws in the soliton background, where F and G vanish, are

$$\delta\psi_{\pm M} = \nabla_M \epsilon_{\pm} \mp \frac{1}{4} H_{MNP} \Gamma^{NP} \epsilon_{\pm} \\ \delta\lambda_{\mp} = \Gamma^M \nabla_M \phi \epsilon_{\pm} \pm \frac{1}{6} H_{MNP} \Gamma^{MNP} \epsilon_{\pm} \quad (5.8)$$

where the subscript $\pm$ denotes the ten-dimensional chirality. Again one finds 16 supertranslational zero modes of the fivebrane solution but, in this case, they transform as two symplectic six-dimensional Majorana–Weyl doublets of the *same* chirality. Thus the non-chiral string theory leads to a chiral static gauge fivebrane, while the chiral string theory leads to a non-chiral static gauge fivebrane!

Worldbrane supersymmetry again implies the existence of four bosonic zero modes in addition to the four translational zero modes. The equation of motion for small fluctuations of the three-form potential C in the soliton background is

$$\nabla^M F_{MNPQ} = \frac{1}{72} \epsilon_{NPQ}{}^{ABCDEFGH} H_{ABC} F_{DEFG}. \quad (5.9)$$

This is solved by

$$C_{MNP} = e^{ik \cdot x} U_{[MN} \nabla_{P]} e^{2\phi}, \quad (5.10)$$

where k is an (anti)-self-dual null-vector in two dimensions tangent to the worldbrane and U is a constant (anti)-self-dual polarization tensor in the four spatial dimensions orthogonal to k and tangent to the worldbrane. In order to obtain a normalizable zero mode, the product $U \wedge k$ must be either self-dual or anti-self-dual, depending on the sign of the axion charge q of the fivebrane. These zero modes lead to an (anti)-self-dual rank-three antisymmetric tensor field strength on the worldbrane, which describes three bosonic degrees of freedom.

We are still short one bosonic zero mode. This can be found in the abelian one-form potential A whose field strength G obeys the equation of motion

$$\nabla^M G_{MN} = \frac{1}{3} F'_{NPQR} H^{PQR}. \quad (5.11)$$

* In components $G_{MN} = 2 \partial_{[M} A_{N]}$, $H_{MNP} = 3 \partial_{[M} B_{NP]}$, $F'_{MNPQ} = 4 \partial_{[M} C_{NPQ]} + 8 A_{[M} H_{NPQ]}$.

This has the zero mode solutions

$$A = e^{ik \cdot x} d e^{-2\phi}, \quad (5.12)$$

$$C = -2 e^{ik \cdot x - 2\phi} H. \quad (5.13)$$

Note that since under abelian gauge transformations $\delta A = d\epsilon$ and $\delta C = -2\epsilon H$, this is equivalent under a “large” gauge transformation to a configuration with vanishing C .

In static gauge, the type IIA fivebrane has one self-dual antisymmetric tensor field, five scalars, and two symplectic Majorana–Weyl doublets of the same chirality. This comprises the $D = 6$ (0, 2) matter supermultiplet.

Finally, we note that there is a closely related solution of eleven-dimensional supergravity which also leads to an effective six-dimensional theory with chiral fermions – in marked contrast to the non-chiral spectrum obtained from compactification of extra dimensions.

6. Duality and the quantization of fivebranes

This section contains some speculative comments on the quantization of fundamental fivebranes and on the possibility of a weak–strong coupling duality in type II string theory.

In special two-dimensional field theories, there is a duality transformation whereby the strongly coupled phase of the theory can be re-expressed as a weakly coupled theory in which the solitons are fundamental particles and the fundamental particles re-appear as solitons. The simplest example of this is the duality between sine–Gordon theory and the massive Thirring model [30]. Montonen and Olive [31] conjectured that an analogous duality occurs two dimensions higher for spontaneously broken gauge theories with the duality interchanging electric and magnetic charge. It was later realized that this conjecture could only be true in $N = 4$ Yang–Mills theory in four dimensions since only then does the magnetic monopole super-multiplet coincide with the gauge super-multiplet [32]. A key aspect of this proposed duality is that, due to the Dirac quantization condition on magnetic charges, the $N = 4$ monopoles are necessarily weakly coupled when the fundamental particles are strongly coupled, and vice versa.

It is natural to ask if such duality transformations could generalize to theories of extended objects in higher dimensions. It was shown in ref. [33], by a straightforward generalization of Dirac’s original argument, that in d dimensions a p -brane is dual in the Dirac sense to a $(d - p - 4)$ -brane. In particular the generalized Dirac quantization condition implies that fivebranes in string theory are weakly coupled at strong coupling, and vice versa. In ref. [1] it was accordingly conjectured

that strongly coupled heterotic string theory might be re-expressed as a theory of weakly coupled fivebranes.

Of course little is known about the problem of quantizing fundamental p -branes (a review can be found in ref. [3]), except for onebranes or strings, because the worldbrane field theories are non-renormalizable for $p > 1$. However the fact that fivebranes arise as soliton solutions of string theory guarantees (assuming the consistency of string theory) that there is a sensible procedure for quantizing excitations of infinitely extended fivebranes. The string field theory functional integral in the soliton sector contains an integration over the modes of the fivebrane, including collective coordinates corresponding to the massless worldbrane fields. If a is the thickness of the fivebrane core, the integration over the modes of the fivebrane amounts to a quantization of the fivebrane with an explicit cutoff a . The presence of an explicit cutoff, of course, avoids the problem of non-renormalizability. At very large distances, the fivebrane dynamics are effectively governed by an action involving only the massless fields of the type discussed in the previous sections.

Difficulty may however arise in attempting to quantize, as is done for strings, excitations of small closed fivebranes. If the typical radius of a fivebrane state is not large compared to its thickness a , it will not be valid to approximate the fivebrane action by the lowest-dimension terms involving the massless fields. Rather one must include all terms in the expansion in the inverse of the cutoff a , and the problem becomes intractable.

It is clear then that establishing the conjecture of ref. [1] for gauge fivebranes will be very difficult. In addition to the basic problem of the quantization of extended objects, the solution of ref. [1] has an arbitrary scale size which makes the existence of a cutoff unclear. Also, even if one could establish the presence of massless states in the spectrum, there are two $N = 1$ supermultiplets in $D = 10$ and one would have to establish the presence of both the gauge and gravity supermultiplets in the spectrum of closed gauge fivebranes.

The situation for the Type II fivebrane seems somewhat more promising as we now discuss. For the type II fivebranes, a can be seen from (2.11) to be $\sqrt{\alpha'}$ – the radius of the semi-wormhole throat. The characteristic radius R of a closed fivebrane can be deduced from the kinetic term of the fivebrane action, which is proportional to

$$(1/\alpha'^3) \int d^6\sigma e^{-2\phi} \eta^{ab} g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu. \quad (6.1)$$

The effective fivebrane tension (with respect to the metric g) is $T_5 = e^{-2\phi}/\alpha'^3$ [1,34] and the characteristic radius is therefore $R \sim e^{\phi/3} \sqrt{\alpha'}$. As strings become more strongly coupled, this radius gets larger, and the effective worldbrane field theory becomes a better and better starting point for computing the spectrum of

closed fivebranes. This lends plausibility to the speculation that a strong coupling expansion of string theory can be developed as a quantization of fundamental fivebranes.

What is the spectrum of a closed fivebrane? To fully characterize the spectrum of excited states (as we can for strings) is probably impossibly difficult, but it may be possible to find the ground state spectrum using topological and symmetry arguments. Since the theory is supersymmetric, the question of whether or not there exist zero energy (with respect to the worldbrane hamiltonian) vacuum states is equivalent to the question of whether or not supersymmetry is spontaneously broken. This might be fruitfully addressed (even in a non-renormalizable theory) by computing the Witten index as in ref. [35], or by low-energy arguments of the type given in sect. 4. If supersymmetry is broken, it is unlikely that there are any zero-mass (in the ten-dimensional sense) states.

On the other hand if worldbrane supersymmetry is unbroken for a closed fivebrane, the ground state is annihilated by all massive fields and non-zero modes of the massless fields. The spectrum can then be determined simply by quantization of the zero modes of the massless fields [36]. In subsect. 5.2, the action governing these modes was described in static gauge. Unfortunately static gauge (unlike lightcone gauge) always breaks down on a closed worldbrane. In addition, because a closed worldbrane is topologically trivial, it is not at all clear that one should expect to find fermion zero modes. Indeed, the natural expectation would be that the lowest fermion modes have energies which are suppressed by powers of $\sqrt{\alpha'}/R$ with R the characteristic size of the worldbrane. Since we have argued that $R \sim e^{\phi/3}\sqrt{\alpha'}$ we should have exact zero modes only as the string coupling becomes infinite. It is not obvious how this should be accounted for in the quantization of a closed fivebrane.

However, depending on the resolution of this issue, if it were possible to construct a covariant action then one might expect for the covariant type II fivebranes that there will be 32 massless worldbrane goldstinos connected with broken spacetime supersymmetry. These transform as two ten-dimensional Majorana–Weyl spinors (just as for the type II string). The zero modes of these fields are the worldbrane part of the space-time supersymmetry charge. The ground state spectrum of the fivebrane must be consistent with the zero mode commutation relations, which is equivalent to requiring that it provide a representation of the ten-dimensional $N = 2$ supersymmetry algebra ^{*}. This suggests that, just as for the type II string, the fivebrane ground state could be given by the massless $D = 10$, $N = 2$ supergravity multiplet.

^{*} A potentially confusing point here is that the chiral (non-chiral) fivebrane is a soliton of the non-chiral (chiral) type II string. This might lead one to believe that quantization of the type IIA (IIB) fivebrane leads to the IIB (IIA) supergravity multiplet. However, we believe this is not the case due to a peculiar relative chirality flip in static gauge, which is detailed in appendix A.

The conjectured duality implies that fundamental strings should appear as solitons of the weakly coupled fivebrane theory (see ref. [37] for a discussion in the context of heterotic string theory). This idea is supported by the following observation. In refs. [38,39] singular supersymmetric solutions to low-energy string theory were found

$$\begin{aligned}
 e^{-2\phi} &= e^{-2\phi_0} + \frac{\alpha'^3}{4\pi^5 x^6}, \\
 g_{\mu\nu} &= \delta_{\mu\nu}, \\
 g_{ab} &= e^{2\phi} \eta_{ab}, \\
 H_{ab\mu} &= -\epsilon_{ab} \nabla_\mu e^{2\phi}.
 \end{aligned} \tag{6.2}$$

These should not be regarded as soliton solutions to string theory, as they do not solve the space-time field-theory equations of motion at the origin. Rather there is a source term at the origin representing a fundamental string. So (6.2) should be interpreted as the field configuration surrounding a fundamental string.

However the statement that these solutions are singular at the origin depends on what metric is being used. The metric in (6.2) is, appropriately, the one which couples to strings in the sigma model. The sigma model (6.1) describing fivebranes however employs the metric $*g = e^{-2\phi}g$. It is easy to see that for the metric $*g$ the configuration (6.2) is not singular, in the sense that the singularity is an infinite distance from any finite point. Thus the singular solutions of ref. [38] are apparently non-singular soliton solutions of the dual fivebrane theory, assuming that this theory exists and has a massless spectrum which is that of $d = 10$, $N = 2$ supergravity. Likewise the smooth solitons of the fundamental string theory representing fivebranes are themselves singular in the fivebrane variables.

7. Conclusions

The purpose of this paper has been to report our preliminary explorations of the dynamics of the fivebrane solitons whose static properties we have discussed in earlier papers. These dynamics are governed by a six-dimensional worldbrane action whose variables are the moduli of the static solution, but whose explicit form is very hard to compute. By using six-dimensional space-time supersymmetry arguments, however, we were able to reach some precise conclusions about the types of term which can be present in the worldbrane effective action. In particular, for the gauge fivebrane, we could show that there is no potential for the scale size of the instanton which forms the core of the soliton and that the existence of a

family of solitons with continuously variable scale-size is an exact property of string theory. The peculiarities of six-dimensional supersymmetry led us to conclude that fivebranes derived from type II string theories must have spin-1 worldsheet degrees of freedom as well as the familiar spin-0 and spin-1/2 degrees of freedom. This is a real novelty and suggests the existence of new class of fundamental p -brane theories. We also offered some remarks on the problem of quantizing fundamental fivebranes. The simplest observation, and one we could not pursue very far, was that our fivebranes come with a built-in worldbrane cutoff which could provide the symmetry-preserving regulator needed to quantize the otherwise non-renormalizable worldbrane action. We were also able, by comparing fivebrane solitons with string solitons, to lend more weight to the conjectured duality between strings and fivebranes and to the notion that, in the strong-coupling limit, the fivebrane action is weakly-coupled and perturbatively quantizable.

We have raised almost as many questions as we have answered, and much more work remains to be done before we can provide any hard results on the quantizability of fivebranes and their duality to strings. We hope to return to the subject in the near future.

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Appendix A. Chirality in static gauge

The super p -brane action with non-linearly realized $N = 1$ ten-dimensional space-time supersymmetry is [17,3]

$$S_{\text{wb}} = \frac{1}{2\pi} \int d^{p+1}\sigma \sqrt{-g} \left(g^{ab} \pi_a^M \pi_b^N \eta_{MN} - p + 1 + ik B_{\text{wzw}} \right), \quad (\text{A.1})$$

$$\pi_a^M = \partial_a X^M - i \bar{\theta} \Gamma^M \partial_a \theta, \quad (\text{A.2})$$

$$M, N = 0, \dots, 10, \quad (\text{A.3})$$

$$a, b = 0, \dots, p \quad (\text{A.4})$$

where $k = \pm 1$, θ is a ten-dimensional Majorana–Weyl spinor, and B_{wzw} is a WZW term. While the expression for B_{wzw} is somewhat complicated for general p , the equation of motion for θ takes the simple form

$$(\mathbb{1} - k\gamma)\gamma^a \partial_a \theta = 0, \quad (\text{A.5})$$

where $\gamma_a \equiv \pi_a^M \Gamma_M$ are the space-time gamma matrices projected onto the world-brane and

$$\gamma \equiv \frac{1}{(p+1)!} \epsilon_{a_1 \dots a_{p+1}} \gamma^{a_1} \dots \gamma^{a_{p+1}} \quad (\text{A.6})$$

obeys $\gamma^2 = \mathbb{1}$. In static gauge

$$X^a = \sigma^a \quad (\text{A.7})$$

the bosonic part of π_a^M is just δ_a^M . The linearized equation of motion for θ is then just

$$P(k) \gamma^a \partial_a \theta = 0, \quad (\text{A.8})$$

where $P(\pm 1)$ are the worldbrane chirality projection operators. Thus only half of the field θ actually propagates on the world-brane and that half has definite worldbrane chirality in static gauge. Which chirality propagates depends only on the sign of k and not on the original ten-dimensional space-time chirality.

For the type II string, S_{wb} contains two terms of the form (A.1) constructed from two space-time spinors, θ_1 and θ_2 . For the IIB string, both spinors have the same space-time chirality, while for the IIA string, they have the opposite chirality. In both cases the sign of k differs for the two terms. Since the static gauge worldsheet chirality depends only on k , it immediately follows that the static gauge action will in both cases contain spinors of both worldsheet chiralities. However, since the transverse $O(8)$ chirality is the product of the worldsheet and space-time chiralities, the static gauge IIA (IIB) string will have spinors of the same (opposite) space-time chirality. Thus the non-chiral type IIA string has transverse chirality in static gauge, while the chiral type IIB string does not.

The above comments probably explain why the IIA theory has a fivebrane soliton with chiral world-brane dynamics and the IIB theory has a fivebrane soliton with non-chiral worldbrane dynamics. Presumably the covariant IIA (IIB) five-brane action, when it is constructed, will involve two terms of the form (A.1) composed of two space-time spinors of opposite (the same) chirality. This chirality assignment is essential for a non-linear realization of the appropriate $N=2$ space-time supersymmetry. The field content and chiralities (both transverse and worldbrane) of the static gauge type IIA (IIB) fivebrane described in subsects. 5.2, 5.1 are then reproduced from the equation of motion (A.8) if the sign of k is the same (opposite) for the two terms. Thus it is plausible that a covariant fivebrane action which non-linearity realizes chiral (non-chiral) ten-dimensional supersymmetry can have non-chiral (chiral) six-dimensional worldbrane supersymmetry in static gauge.

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