

COSMIC STRINGS AS ORBIFOLDS

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We construct string theory versions of cosmic strings by considering orbifold compactifications of spacetime down to two dimensions.

1. Cosmic strings

In this paper we shall discuss a variety of problems that arise in string theory when spacetime is compactified down to two, rather than four, dimensions. One motivation for our work is that it allows us to construct string theory versions of *cosmic* strings. We will see that such cosmic strings probably have too large an energy density to be of direct astrophysical relevance, but a description of how we reach this conclusion is a good way to introduce and motivate the problems studied in the rest of the paper.

Recall that in four dimensions a cosmic string of tension μ and thickness zero, running along the z -axis, is described by an energy-momentum tensor of the form [1]

$$T^i_j = \begin{pmatrix} \mu & & & \\ & 0 & & \\ & & 0 & \\ & & & \mu \end{pmatrix} \delta(x) \delta(y). \quad (1)$$

The corresponding solutions to Einstein's equations describe a very simple geometry. The spacelike sections transverse to the z -axis are cones, with deficit angles

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$\Delta = 8\pi G\mu$. The cones are stacked together so that spacetime is flat – except, of course, at the tips of the cones, which correspond to the location of the string. The manifold has nontrivial holonomy, since transport around the string rotates a vector through an angle Δ about the z -axis. Each of the transverse cones is equivalent to an angular sector of a plane, of opening angle $2\pi - \Delta$. The points on the boundaries are identified up to a tangent space rotation of angle Δ .

The fact that conical spaces are solutions to Einstein's equations is peculiar to four dimensions. In higher dimensions, a source such as (1) does not lead to such a simple geometry. The appropriate higher-dimensional generalizations of conical spaces are the orbifolds discussed in ref. [2]. In this paper we will construct orbifold compactifications of string theory that have a natural interpretation in terms of cosmic strings. Our solutions, at least at the classical level, correspond to infinitely thin strings. They are different from usual cosmic strings in that they have no obvious topological interpretation. We will find various consistency conditions, but one should remember that our restrictions are not necessarily limitations on physical strings of nonzero thickness – we only consider the special case of infinitely thin strings for which we can find exact solutions of the classical equations of motion.

To understand the relation between orbifolds and cosmic strings, we shall first consider string propagation on conical spacetimes. Since cones are asymptotically flat, there is no question that strings can consistently propagate far from the cosmic string source. Therefore we only need examine what happens when a string encounters the singularity at the apex of a cone. For general Δ it appears that the string cannot propagate across the singularity without developing a kink, an event which is probably too singular for consistent propagation. For special values of Δ , however, the string can flow smoothly across the singularity. To see this, let us consider a string propagating on a sector of the plane of opening angle $2\pi - \Delta$. If the string passes through one boundary, it must re-emerge at the corresponding point on the other boundary, rotated through the holonomy angle Δ (see fig. 1). This is no problem when the opening angle is $2\pi/N$. Then N copies of the cone exactly cover the plane, and N copies of a string, each rotated by $2\pi/N$, give a consistent string configuration on the plane. Conversely, string configurations on the plane, symmetric under $2\pi/N$ rotations, are also consistent string configurations on a cone of deficit angle $\Delta = 2\pi - 2\pi/N$, as can be seen by “modding out” the plane with respect to $2\pi/N$ rotations about the z -axis. Such N -fold symmetric configurations



Fig. 1. Closed strings on a cone of deficit angle $\Delta = \frac{4}{3}\pi$. The solid and dashed strings have winding numbers zero and one, respectively.

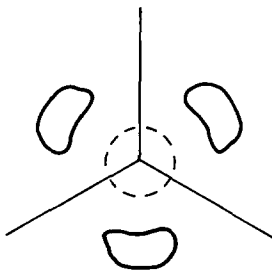


Fig. 2. For deficit angles $\Delta = 2\pi - 2\pi/N$, the N -fold symmetric configurations on a plane correspond to consistent string configurations on a cone.

automatically satisfy the correct boundary conditions at the edge of each angular slice (see fig. 2).

Thus, free string propagation on the plane can be turned, by a symmetry restriction, into the nonsingular propagation of free strings on cones of special deficit angles, $\Delta = 2\pi - 2\pi/N$. An important feature of string configurations on the cone is the existence of winding number sectors with respect to the singular point at the apex of the cone (see figs. 1 and 2). With the above understanding of how strings propagate across the singular point, it is clear that the winding number is only conserved modulo N on a cone of angular size $2\pi/N$. This leads to N “twisted sectors” of strings on the cone, distinguished by the independent closed string boundary conditions $X^i(\pi) = R^{ij}(n\Delta)X^j(0)$, where $n = 1, \dots, N$, and $R^{ij}(n\Delta)$ is a rotation through angle $n\Delta$.

The above line of argument is obviously very closely related to the orbifold method of constructing nontrivial spacetime compactifications [2]. It indicates that the singular points of the orbifold can, in the right circumstances, be interpreted as cosmic strings of special deficit angles. Since the deficit angles produced in this manner are large (at least equal to π), their direct phenomenological relevance is questionable, but we shall find it very instructive to pursue their study.

The vacuum energy of a cosmic string has a volume term (which is proportional to the flat space cosmological constant) and a volume-independent term, which, in this context, should be interpreted as a string loop correction to the tension of the cosmic string. This correction is a very interesting quantity in its own right, and important to calculate in order to check modular invariance. To do this, we must put the string in a finite transverse “box”, and study the dependence of the vacuum energy on the box size. Since we are trying to compute the effect of boundary conditions on the vacuum energy, it is essential not to impose some new kind of boundary condition on the space transverse to the string. This leads us to construct our cosmic strings by an exact copy of the orbifold method. We start with a two-dimensional torus, which is by definition compact, and we divide it by an automorphism of the lattice that defines the torus. This produces a space with

several singular points which we can interpret as a collection of cosmic strings, disposed in just such a way that their transverse spaces join together to produce a compact space. Since the only “boundaries” of this manifold are provided by the cosmic strings themselves, the correction to the cosmic string tension can be unambiguously determined from the vacuum energy.

In short, to study cosmic strings, one should use orbifolds to compactify four-dimensional space down to two dimensions. Remembering that there already is a six-dimensional internal space, we see that we really want to study orbifold compactification from ten dimensions down to two, in such a way that six of the compactified dimensions are “small” and two are “big”. Such a manifold has holonomy H contained in $O(6) \times O(2) \subset O(8)$. Our remarks have indicated that such compactifications are possible at the classical level, and we now want to see what complications arise at the quantum level. We shall find that consistent compactifications are possible when $H \subseteq SU(3) \times U(1) \subset SU(4)$. Furthermore, we shall see that a number of interesting technical issues arise that were not apparent in studies of six-dimensional compactifications.

2. Sigma models with $SU(4)$ holonomy

In the previous section we have seen that the question of superstring propagation on cosmic string backgrounds leads to the study of string propagation on eight-dimensional orbifolds. In this section we will examine more general properties of eight-dimensional compactifications.

Superstring propagation on an arbitrary background manifold is described by a two-dimensional supersymmetric nonlinear sigma model. The target space of the sigma model corresponds to the background spacetime manifold. For compactification down to two dimensions, appropriate background manifolds are of the form $\mathcal{M}_2 \times K$, where $\mathcal{M}_2$ is two-dimensional Minkowski space, and K is an eight-dimensional compact manifold. In the light-cone gauge, the target space of the sigma model is given by the manifold K .

Perhaps the simplest compactification is to take the manifold K to be the eight-torus T^8 . In this case the supersymmetric sigma model takes the following form:

$$\mathcal{L} = -\frac{1}{2} \partial_\alpha X^i \partial^\alpha X^i - \frac{1}{2} \bar{S}^a \rho^\alpha \partial_\alpha S^a. \quad (2)$$

The X^i are the coordinates of the torus T^8 , and the S^a are world-sheet spinors, where a runs from 1 to 8. (World-sheet spinor indices are suppressed.) If we take the S^a to transform as a spinor of $O(8)$, the lagrangian (2) describes a closed (type II) superstring propagating on $\mathcal{M}_2 \times T^8$, in the Green-Schwarz formalism.

The above lagrangian is invariant under two sets of supersymmetry transformations. The first set is linear,

$$\begin{aligned}\delta X^i &= \bar{\epsilon}^{\dot{a}} \gamma_{\dot{a}a}^i S^a, \\ \delta S^a &= \partial_\alpha X^i \gamma^{ia\dot{a}} \rho^\alpha \epsilon^{\dot{a}},\end{aligned}\tag{3}$$

where the $\epsilon^{\dot{a}}$ are eight constant world-sheet spinors, of opposite chirality to the S^a . The ϵ -spinors carry dotted indices to denote the fact that they are also O(8) spinors, of opposite O(8) chirality to S^a . It is easy to check that two ϵ -supersymmetries close into a world-sheet translation.

The lagrangian (2) possesses a second supersymmetry,

$$\begin{aligned}\delta X^i &= 0, \\ \delta S^a &= \eta^a.\end{aligned}\tag{4}$$

Here the η^a are eight constant world-sheet spinors, of the *same* chirality as S^a . As is evident from (4), they also form an O(8) spinor of the same chirality as S^a . Two ϵ - and η -supersymmetries close into a spacetime translation.

The spinors $\epsilon^{\dot{a}}$ and η^a generate independent symmetries of the light-cone action. On $\mathcal{M}_{10}$, they combine to parametrize two ten-dimensional spacetime supersymmetries. For compactifications on $\mathcal{M}_2 \times T^8$, the spinors $\epsilon^{\dot{a}}$ and η^a label a (16,16) supersymmetry of the two-dimensional *spacetime* spectrum.

On the torus T^8 , the string spectrum does not depend on the O(8) chirality of S . This stems from the fact that the holonomy of T^8 is trivial – the sigma model does not know whether S is left- or right-handed. On more complicated manifolds, the story is more subtle. The holonomy group distinguishes between 8_L and 8_R , and one must be careful.

To see what happens in a more general case, let us return for a moment to the lagrangian (2). Since we are describing closed strings, left- and right-moving fermions never mix. They split naturally into *independent* left- and right-moving spinor representations of O(8). If they are assigned O(8) representations of *opposite* chirality, the two-dimensional spectrum is guaranteed to be non-chiral. If, however, the left- and right-movers are given the *same* O(8) chirality, the spectrum has the possibility to be chiral. This case is much more interesting, and is the one on which we focus in this paper.

A particularly important class of compactifications is found when K is Kähler and Ricci-flat. Such manifolds are the starting point for constructing one set of consistent string backgrounds, order-by-order in the string tension α' [4]. These manifolds have SU(4) holonomy, and are the eight-dimensional generalizations of the Calabi-Yau spaces discussed in ref. [3].

Manifolds of SU(4) holonomy are best described in terms of complex coordinates. The eight real coordinates X^i become four complex coordinates Z^i and four

conjugates $\bar{Z}^{\bar{j}}$. The spinors S^a should also be described in terms of the SU(4) holonomy. This is because the left- and right-handed spinors of O(8) have *different* decompositions under SU(4):

$$\begin{aligned} 8_L &\rightarrow 4 + \bar{4}, \\ 8_R &\rightarrow 6 + 1 + 1. \end{aligned} \quad (5)$$

The holonomy group distinguishes between the 8_L and 8_R , so the corresponding nonlinear sigma models depend on the O(8) chirality of S^a . One of the models is similar to those previously considered, with the vector and the spinor both transforming as a $4 + \bar{4}$ of SU(4). The other model is completely different, with the vector and the spinor transforming in different representations of the holonomy group. As we shall see, both models are necessary to describe the physics of superstring compactifications down to two dimensions.

To understand the roles of the two sigma models, let us first recall that the light-cone sigma models are found by boosting the covariant Green-Schwarz action [5] to the infinite-momentum frame. On manifolds $\mathcal{M}_2 \times K$, there are two choices for this boost: either parallel or antiparallel to the z -axis. Boosting parallel to the z -axis singles out the up-moving states. It also eliminates half of each sixteen-dimensional O(9,1) spinor. Boosting antiparallel to the z -axis gives the down-moving states. Furthermore, it annihilates the other half of the O(9,1) spinors. For the case at hand, this implies that the sigma model based on the 8_L decomposition describes the up-moving states, while the sigma model based on 8_R describes their down-moving counterparts.

On the manifold T^8 , there is no difference between the 8_L and 8_R , so the up- and down-moving states are the same. This is one way to see that string compactification on $\mathcal{M}_2 \times T^8$ gives a nonchiral spectrum. On manifolds of SU(4) holonomy, the decomposition (5) implies that the up- and down-moving states are described by *different* sigma models. They give rise to different spectra, and the compactifications are inherently chiral.

To study the spectra, we must first construct the up- and down-moving sigma models. The up-moving model is defined by the decomposition $8_L \rightarrow 4 + \bar{4}$. In an SU(4) basis, the eight real spinors S^a become four complex spinors S^i and four conjugate spinors $\bar{S}^{\bar{j}}$. The supersymmetric nonlinear sigma model takes the following form:

$$\mathcal{L} = -g_{i\bar{j}} \partial_\alpha Z^i \partial^\alpha \bar{Z}^{\bar{j}} - g_{i\bar{j}} \bar{S}^{\bar{i}} \rho^\alpha D_\alpha S^{\bar{j}} + \frac{1}{3} R_{i\bar{j}k\bar{l}} \bar{S}^{\bar{i}} S^{\bar{j}} \bar{S}^{\bar{k}} S^{\bar{l}}. \quad (6)$$

Here $D_\alpha S^i = \partial_\alpha S^i + \Gamma_{jk}^i \partial_\alpha Z^j S^k$ is the SU(4) covariant derivative. Since $8_R \rightarrow 6 + 1 + 1$, the manifold K admits two covariantly constant spinors, of opposite chirality to S^a . These spinors combine to generate a linear supersymmetry of the

lagrangian (6):

$$\begin{aligned}\delta Z^i &= \bar{\epsilon} S^i, \\ \delta S^i &= \partial_\alpha Z^i \rho^\alpha \epsilon - \Gamma_{jk}^i \delta Z^j S^k,\end{aligned}\tag{7}$$

where ϵ is a (complex) covariantly-constant world-sheet spinor. As before, two ϵ -supersymmetries close into a constant world-sheet translation. Note that (6) has no η -supersymmetry since the 8_L decomposition of the 8_L contains no singlets.

The down-moving sigma model is based on the decomposition $8_R \rightarrow 6 + 1 + 1$. The eight real spinors S^a become six real spinors $S^{[ij]}$, plus one complex spinor S . The corresponding sigma model must be of the form

$$\mathcal{L} = -g_{ij} \partial_\alpha Z^i \partial^\alpha \bar{Z}^j - \frac{1}{2} \bar{S}_{ij} \rho^\alpha D_\alpha S^{ij} - \bar{S} \rho^\alpha \partial_\alpha S + \frac{1}{12} R_{ik}^j \bar{S}^{im} S_{mj} \bar{S}^{kn} S_{nl}.\tag{8}$$

As before, the derivative D_α is covariant with respect to $SU(4)$. The lagrangian (8) has a nonlinear supersymmetry:

$$\begin{aligned}\delta X^i &= 0, \\ \delta S^{ij} &= 0, \\ \delta S &= \eta,\end{aligned}\tag{9}$$

where η is a complex, covariantly constant world-sheet spinor. It does not, however, have a linear supersymmetry ϵ .

What do these models tell us about the full string spectrum? The up-moving lagrangian is invariant under a linear supersymmetry. This forces the massive string states to be manifestly supersymmetric. The massless states, however, need not occur in Bose-Fermi pairs. They are described by the zero-energy states of the sigma model. The zero-energy states form supersymmetry singlets, and are annihilated by the supercharge. This leads us to expect that the massive up-moving spectrum will be supersymmetric, while the massless up-moving states will not.

The down-moving lagrangian is invariant under the nonlinear transformations of eq. (9). These transformations describe a system with a spontaneously broken supersymmetry, where the spinor S plays the role of the Goldstone fermion. This implies that the spectrum of the down-movers is completely supersymmetric: For every state of a given energy, there is a second state of the same energy, with an extra Goldstone fermion. Taken together, the two sigma models describe a space-time system with $(0, 4)$ supersymmetry. (The supersymmetry counting can be extended to arbitrary holonomy $H \subseteq SU(4)$. Suppose that the 8_L has p singlets under H , and that the 8_R has q . Then the ϵ and η supersymmetries combine to give a $(2p, 2q)$ spacetime supersymmetry in $1 + 1$ spacetime dimensions. In a similar fashion, it is not hard to show that nonchiral strings have a $(p + q, p + q)$ spacetime

supersymmetry, and that the heterotic string has at least a $(p+q,0)$ spacetime supersymmetry.)

The up- and down-moving spectra come from different sigma models, but they are not completely independent. The string remembers that the two sigma models describe the same spacetime system. Two-dimensional Lorentz invariance ensures that the massive up- and down-moving spectra are identical. However, two-dimensional Lorentz invariance is not enough to ensure that the massless states are identical as well. Massless up- and down-moving states are not related by any symmetries, so the massless spectra are free to differ – as indeed they do. It is precisely this freedom that gives rise to chiral superstring compactifications down to two dimensions. The fact that the massive states are supersymmetric implies that certain amplitudes receive contributions from only the massless states. In the following section, we will discuss the interpretation and consistency of this result, with regard to some specific examples.

3. Particular orbifolds

Having presented the general features of compactification down to two dimensions, we will now examine several models in more detail. We will use light-cone quantized string theory, with eight spatial dimensions compactified on different orbifolds K . We construct K by modding out an eight-torus T^8 with respect to a crystallographic point group P , so $K = T^8/P$. If we write $T^8 = T^2 \times T^6$, where we distinguish between two large “external” dimensions and six small “internal” ones, we find four-dimensional spacetimes with cosmic-string-like singularities.

Virtually all of the physics of the orbifold is governed by the choice of the point group P . For simplicity, we consider abelian point groups Z_N , where Z_N lies in $O(6) \times O(2)$. Furthermore, to avoid tachyons far from the string, the group elements must lie in $SU(3) \times U(1) \subset U(4)$. It will therefore be useful to put complex coordinates z^0, z^1, z^2, z^3 on the eight-dimensional manifolds. For definiteness, let z^0 describe the two external directions transverse to the cosmic strings, and let the three other complex coordinates describe the six internal dimensions. The twists we consider are products of independent rotations in the planes described by the four complex coordinates. They can be defined by the four phases that characterize the rotations:

$$g = \text{diag}(e^{i\phi_0}, e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}). \quad (10)$$

Note that if the product of the four phases is unity, the holonomy matrix is really an element of $SU(4)$. It is a well-known feature of string theory that manifolds of $SU(3)$ holonomy have very favorable features, and one of our points will be that the same is true for $SU(4)$ holonomy.

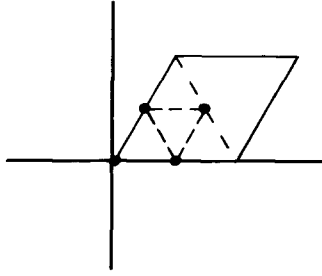


Fig. 3. Dividing a two-torus by Z_2 gives an orbifold construction of a tetrahedron.

In this section we consider the following four choices for the point group Z_N :

Case	Group	Generators
1	Z_2	$g = (-1, 1, 1, 1)$
2	Z_2	$g = (-1, -1, -1, -1)$
3	$Z_2 \otimes Z_2$	$g = (-1, -1, 1, 1), (1, 1, -1, -1)$
4	Z_6	$g = (-1, e^{i\pi/3}, e^{i\pi/3}, e^{i\pi/3})$

In case 1 the point group acts as an inversion on the external two-torus, leaving the internal six-torus alone. This is the simplest version of a cosmic string. It produces an orbifold with four singular points, each of deficit angle π . This orbifold can be visualized as a tetrahedron (see fig. 3). The full spacetime is simply this tetrahedron tensored with the internal six-torus, and amounts to four cosmic strings, each of deficit angle π . Note that the holonomy of this example is not in $SU(4)$. We will find, not coincidentally, that this model has a tachyon.

The remaining three cases are all of $SU(4)$ holonomy, and will turn out to be tachyon-free. We will use them to illustrate problems that arise in the direct compactification of ten dimensions down to two via manifolds of $SU(4)$ holonomy. Case 2 will be used to discuss string loop corrections to cosmic string backgrounds. We will compute the one-loop partition function, and argue that it corresponds to a renormalization of the tension μ of the cosmic string. Case 3 can be viewed as an orbifold limit of string compactification on $K3 \otimes K3$. It will be used to illustrate the connection between string and field theory compactifications. Case 4 is a model of a cosmic string built on a semi-realistic string vacuum, as can be seen by considering

$$g^2 = \text{diag}(1, e^{i2\pi/3}, e^{i2\pi/3}, e^{i2\pi/3}) \quad (11)$$

and the other two elements of the Z_3 group it generates. These elements leave the two external dimensions untouched and generate an internal six-dimensional orbifold. This internal orbifold is known to yield a quasi-realistic spectrum for the

heterotic string [2]. The singular points of the full orbifold are the fixed points of the group elements g^n , for $n = 1, \dots, 6$. For n even, these fixed points are in the internal space. For n odd, there is a single fixed point which lies at the origin of the eight-torus. It gives a singularity that can be interpreted as a single cosmic string. Since g acts on all dimensions, the cosmic string has a nontrivial internal structure which should be apparent in its excitation spectrum.

We now calculate the spectrum of these models to see whether they make quantum-mechanical sense. We will use the Green-Schwarz method of quantizing the closed superstring in light-cone gauge. Recall that the Green-Schwarz string contains bosonic and fermionic coordinates in eight-dimensional representations of the transverse Lorentz group $O(8)$. The bosons are in the vector 8_V , while the fermions are in the spinor 8_L or 8_R . In general, there is physics in this choice, for it determines how the fermions respond to the holonomy group. As described in the previous section, one choice describes the physics of the “up” light-cone, and the other that of the “down.”

To see how this works, recall that the choice of $O(8)$ representation may be made independently for the left- and right-moving spinors on the string world-sheet. This gives four possible choices, which correspond to the up- and down-moving sectors of two closed Green-Schwarz strings:

non-chiral	up-moving	$\tilde{S}^a \in 8_L$	$S^{\dot{a}} \in 8_R$
	down-moving	$\tilde{S}^{\dot{a}} \in 8_R$	$S^a \in 8_L$
chiral	up-moving	$\tilde{S}^a \in 8_L$	$S^a \in 8_L$
	down-moving	$\tilde{S}^{\dot{a}} \in 8_R$	$S^{\dot{a}} \in 8_R$

Here we have denoted the left-moving spinors by $\tilde{S}$, and their right-moving counterparts by S . As in sect. 2, the 8_L and 8_R are denoted by undotted and dotted spinor indices.

3.1. TETRAHEDRON $\otimes T^6$

Let us restrict our attention to the chiral superstring and examine the orbifold characterized by $g = (-1, 1, 1, 1)$. This element generates a rotation in a two-dimensional subspace, so it is appropriate to decompose the holonomy group according to $O(8) \rightarrow O(6) \times O(2)$, or equivalently, $O(8) \rightarrow SU(4) \times U(1)$. The vector and spinor representations then decompose according to

$$\begin{aligned}
 8_V &\rightarrow 6(0) + 1(1) + 1(-1), \\
 8_L &\rightarrow 4\left(\frac{1}{2}\right) + \bar{4}\left(-\frac{1}{2}\right), \\
 8_R &\rightarrow 4\left(-\frac{1}{2}\right) + \bar{4}\left(\frac{1}{2}\right).
 \end{aligned} \tag{12}$$

The numbers in parentheses are the $U(1)$ charges: an entry (α) means that the corresponding component is multiplied by $e^{i\pi\alpha}$ under the action of g . The decomposition of the 8_V expresses the fact that the inversion leaves six of the eight bosonic coordinates alone and changes the sign of the other two. The spinor decomposition shows that, under the inversion, four of the fermionic coordinates are multiplied by i , and the other four by $-i$. Note that since the 8_L and 8_R carry the same $U(1)$ charges, the up- and down-light-cones are equivalent.

Since the holonomy group is Z_2 , the spectrum contains a twisted and an untwisted sector. The spectrum of the untwisted sector is found by projecting the standard Green-Schwarz states onto those that are invariant under g . The states of the twisted sector are obtained by quantizing the string subject to the twisted boundary condition $X^i(\pi) = g \cdot X^i(0)$, $S^a(\pi) = g \cdot S^a(0)$. This boundary condition implies that the creation and annihilation operators of the twisted fields are no longer integer-moded. For this choice of g , two of the bosons are $Z \pm \frac{1}{2}$ moded, and all fermions are $Z \pm \frac{1}{4}$ moded. The mass-squared of the ground state is the sum of the normal ordering constants for the bosons, minus the sum of the normal-ordering constants for the fermions. Since the twist treats the bosons and fermions asymmetrically, this quantity need not vanish. For the case at hand, the vacuum energy turns out to be $-\frac{1}{4}$. (To see this, note that the normal-ordering constant for a real η -moded bosonic oscillator is $-\frac{1}{4}(\eta^2 - \eta + \frac{1}{6})$.) This means that the twisted-sector ground state is a tachyon, and the model is unphysical.

3.2. Z_2 ORBIFOLD

The remaining three models also treat bosons and fermions asymmetrically, but because they have $SU(4)$ holonomy, they do not suffer from tachyons. Let us first consider the orbifold defined by $g = (-1, -1, -1, -1)$. This group element is in the center of the $SU(4)$ that acts on the four complex coordinates $z^0 \dots z^3$, so it is again appropriate to decompose representations according to $O(8) \rightarrow SU(4) \times U(1)$. Because we are considering a different embedding of $SU(4)$ in $O(8)$ than in the preceding case, we have a different result for the vector and spinor decompositions:

$$\begin{aligned} 8_V &\rightarrow 4 + \bar{4}, \\ 8_L &\rightarrow 4 + \bar{4}, \\ 8_R &\rightarrow 6 + 1 + 1. \end{aligned} \tag{13}$$

Under the action of g , the 8_V and 8_L change sign, while the 8_R is unaffected.

Since the 8_L and the 8_R respond differently to the Z_2 twist, this model is inherently chiral. The up light-cone is characterized by $(X^i, \tilde{S}^a, S^a) = (8_V, 8_L, 8_L)$, while the down light-cone is defined by $(X^i, \tilde{S}^a, S^a) = (8_V, 8_R, 8_R)$. The up light-cone treats bosons and fermions symmetrically, while the down light-cone does not.

TABLE 1
 Z_2 orbifold: The untwisted, up-moving spectrum

Mass	Left-movers		Right-movers	
	parity +	parity -	parity +	parity -
0	$ \dot{\tilde{a}}\rangle$	$ \tilde{i}\rangle$	$ \dot{a}\rangle$	$ i\rangle$
1	$\tilde{\alpha}_{-1}^i \tilde{j}\rangle$ $\tilde{S}_{-1}^a \tilde{i}\rangle$	$\tilde{\alpha}_{-1}^i \dot{\tilde{a}}\rangle$ $\tilde{S}_{-1}^a \dot{\tilde{b}}\rangle$	$\alpha_{-1}^i j\rangle$ $S_{-1}^a i\rangle$	$\alpha_{-1}^i \dot{a}\rangle$ $S_{-1}^a \dot{b}\rangle$

Let us compute the states of the string through the first mass level. We start with the up-movers. In the untwisted sector, the bosons and the fermions obey periodic boundary conditions. They are integer-moded, and the ground state lies at zero energy. There are eight left-moving and eight right-moving fermionic zero modes, so the vacuum represents the $O(8)$ Clifford algebra. Since the zero modes transform as 8_L of $O(8)$, the bosonic vacua $|\tilde{i}\rangle$ and $|i\rangle$ transform as 8_V , while the spinor vacua $|\dot{\tilde{a}}\rangle$ and $|\dot{a}\rangle$ form two 8_R . The excited states are built on these vacua with the integer-moded creation operators $\tilde{\alpha}_{-n}^i$, α_{-n}^i , $\tilde{S}_{-n}^a$ and S_{-n}^a , with $n > 0$. The resulting spectrum is shown in table 1.

The states in table 1 are labelled by their $O(8)$ representations. However, for compactifications on $\mathcal{M}_2 \times K$, the $O(8)$ representations simply count the number of two-dimensional states. $O(8)$ spinor representations label spacetime fermions, while $O(8)$ tensor representations count the number of bosons. The physical states are found by tensoring the left-movers with the right-movers at the same mass level, and then picking out the Z_2 -invariant states. In the untwisted sector, this gives a restriction on the internal momenta. States of even oscillator parity are restricted to even-parity momenta, while those of odd oscillator parity are required to have odd momenta.

In the twisted sector, the fermions and the bosons both obey antiperiodic boundary conditions. The vacuum is still at zero energy, and is unique because there

TABLE 2
 Z_2 orbifold: The twisted, up-moving spectrum

Mass	Left-movers		Right-movers	
	parity +	parity -	parity +	parity -
0	$ 0\rangle$		$ 0\rangle$	
$\frac{1}{2}$		$\tilde{\alpha}_{-1/2}^i 0\rangle$ $\tilde{S}_{-1/2}^a 0\rangle$		$\alpha_{-1/2}^i 0\rangle$ $S_{-1/2}^a 0\rangle$
1	$\tilde{\alpha}_{-1/2}^i \tilde{\alpha}_{-1/2}^j 0\rangle$ $\tilde{\alpha}_{-1/2}^i \tilde{S}_{-1/2}^a 0\rangle$ $\tilde{S}_{-1/2}^a \tilde{S}_{-1/2}^b 0\rangle$		$\alpha_{-1/2}^i \alpha_{-1/2}^j 0\rangle$ $\alpha_{-1/2}^i S_{-1/2}^a 0\rangle$ $S_{-1/2}^a S_{-1/2}^b 0\rangle$	

TABLE 3
 Z_2 orbifold: The untwisted, down-moving spectrum

Mass	Left-movers		Right-movers	
	parity +	parity -	parity +	parity -
0		$ \tilde{i}\rangle$ $ \tilde{a}\rangle$		$ i\rangle$ $ a\rangle$
1	$\tilde{\alpha}_{-1}^i \tilde{j}\rangle$ $\tilde{\alpha}_{-1}^i \tilde{a}\rangle$	$\tilde{S}_{-1}^{\tilde{a}} \tilde{i}\rangle$ $\tilde{S}_{-1}^{\tilde{a}} \tilde{b}\rangle$	$\alpha_{-1}^i j\rangle$ $\alpha_{-1}^i a\rangle$	$S_{-1}^a i\rangle$ $S_{-1}^a b\rangle$

are no bosonic or fermionic zero modes. The massive states are constructed off the vacuum with half-odd-integer creation operators. The resulting spectrum appears in table 2. As before, the physical states are found by tensoring the left- and right-movers. In the twisted sector, there are no internal momenta, so the Z_2 projection annihilates all states with odd oscillator parity.

We now consider the down-movers. In the untwisted sector, all fields obey periodic boundary conditions, so the vacuum energy is again zero. The states are the same as in table 1, rearranged as in table 3. (Since the zero modes are now in 8_R , the spinor vacua are changed to 8_L . From the two-dimensional point of view, they still contain eight fermions. However, since $|\tilde{a}\rangle, |a\rangle \in 8_L$, the vacua transform by a sign under the Z_2 transformation.)

In the twisted sector, the bosons are half-odd-integer moded, while the fermions retain their integer moding. This gives a vacuum energy of $+\frac{1}{2}$, with a 16-fold Bose-Fermi vacuum degeneracy because of the fermion zero modes. The states are collected in table 4; the physical states are found as before.

Let us pause for a moment to compare the orbifold spectrum with the general expectations discussed in sect. 2. We first note that the up- and down-moving massive spectra are identical. This is obvious at mass level $\frac{1}{2}$, but is less so at higher levels. Of course, this mass equality is a necessary consequence of two-dimensional

TABLE 4
 Z_2 orbifold: The twisted, down-moving spectrum

Mass	Left-movers		Right-movers	
	parity +	parity -	parity +	parity -
0				
$\frac{1}{2}$		$ \tilde{i}\rangle$ $ \tilde{a}\rangle$		$ i\rangle$ $ a\rangle$
1	$\tilde{\alpha}_{-1/2}^i \tilde{j}\rangle$ $\tilde{\alpha}_{-1/2}^i \tilde{a}\rangle$		$\alpha_{-1/2}^i j\rangle$ $\alpha_{-1/2}^i a\rangle$	

Lorentz invariance, and provides a useful check of our results. (We have verified that the equality holds for all levels by computing the up- and down-moving partition functions.) The massless spectra, however, are *not* identical. There is no problem with this, for up- and down-moving massless states are not related in two dimensions. Note that the down-moving spectrum is completely supersymmetric, at both the massive and massless levels. In contrast, the up-moving spectrum is only partially supersymmetric. The massive states are supersymmetric – as required by Lorentz invariance – but the massless states are not. The Z_2 orbifold describes a chiral system with $(0,4)$ supersymmetry, as discussed in sect. 2.

Now that we have the physical spectrum, it is easy to calculate the one-loop correction to the vacuum energy. For compactifications down to two dimensions, it is given by

$$\begin{aligned}\Lambda_{\text{one-loop}} &= \frac{1}{2} \text{Str} \ln \Delta^{-1} \\ &= -\frac{1}{(2\pi)^2} \int_F \frac{d^2\tau}{(\text{Im } \tau)^2} \text{Str } q^{L_0+h_0} \bar{q}^{\bar{L}_0+\bar{h}_0},\end{aligned}\quad (14)$$

where $q = e^{2\pi i\tau}$, L_0 and $\bar{L}_0$ are the left- and right-moving Virasoro generators, and h_0 and $\bar{h}_0$ are the corresponding vacuum energies. The integral $d\tau$ spans one fundamental region F of the modular group.

For the Z_2 orbifold, we found that the massive states were completely supersymmetric. Thus they give no contribution to the vacuum energy. The entire contribution to $\Lambda_{\text{one-loop}}$ comes from the massless states,

$$\begin{aligned}\Lambda_{\text{one-loop}} &= -\frac{1}{(2\pi)^2} \int_F \frac{d^2\tau}{(\text{Im } \tau)^2} (N_B - N_F)|_{E=0} \\ &= -\frac{1}{12\pi} (N_B - N_F)|_{E=0}.\end{aligned}\quad (15)$$

Note that the integrand in (15) is modular invariant, despite the fact that it receives contributions from only the massless states. This is a surprising but generic feature of compactification on manifolds of $SU(4)$ holonomy. Modular invariance is a necessary, nontrivial check on the quantum consistency of the string theory, and manifolds of $SU(4)$ holonomy pass with flying colors. A second important point is that $\Lambda_{\text{one-loop}}$ is independent of the size of the compact space K . (For an orbifold, it is independent of the size of the transverse torus T^8 .) Physically, this implies that $\Lambda_{\text{one-loop}}$ does not renormalize the cosmological constant – the energy density far from the string remains unchanged. Instead, $\Lambda_{\text{one-loop}}$ gives a string loop correction to the energy density μ of the string itself. Such a renormalization of μ should feed back to change the conical geometry of spacetime – an issue to which we shall return in sect. 4.

The final issue we would like to discuss with regard to the Z_2 orbifold is the equivalence of the Green-Schwarz and Neveu-Schwarz-Ramond formalisms. Up to now we have only considered the Green-Schwarz formalism, where the world-sheet fermions transform in a spinor representation of $O(8)$. In the Neveu-Schwarz-Ramond formalism, the world-sheet fermions form a *vector* representation of $O(8)$. On a flat space or on a manifold of $SU(3)$ holonomy, the equivalence of the two formalisms is a consequence of $O(8)$ triality – the holonomy group cannot distinguish between $O(8)$ representations related by triality. On a manifold of $SU(4)$ holonomy, the equivalence is less clear, for the $O(8)$ triality is broken. In the appendix we compute the spectrum of the Z_2 orbifold using the Neveu-Schwarz-Ramond formalism. We show that the spectrum is identical to that computed above, despite the lack of triality. The GS-NSR equivalence appears to hold even on manifolds of $SU(4)$ holonomy.

3.3. $Z_2 \otimes Z_2$ ORBIFOLD

We now turn to the $Z_2 \otimes Z_2$ orbifold. Since dividing a four-torus by Z_2 gives a singular limit of the space $K3$, this orbifold describes a singular limit of compactification on $K3 \otimes K3$. Compactification on $K3$ leads to chiral fermions, so this is a particularly interesting case for comparing field theory and orbifold compactifications.

The manifold $K3$ is the four-dimensional analogue of a Calabi-Yau space. It is Kähler with $SU(2)$ holonomy, and in the orbifold limit, the holonomy group is the center of $SU(2)$. Since $K3 \otimes K3$ is a product space, the obvious decomposition for the holonomy group is $O(8) \rightarrow SU(2) \times SU(2) \times SU(2) \times SU(2)$, where we ignore two of the $SU(2)$ factors. This gives the following branchings for the vector and spinor representations of $O(8)$:

$$\begin{aligned} 8_V &\rightarrow 2(2, 1) + 2(1, 2), \\ 8_L &\rightarrow 2(2, 1) + 2(1, 2), \\ 8_R &\rightarrow 4(1, 1) + (2, 2). \end{aligned} \tag{16}$$

The numbers inside the parentheses identify the two $SU(2)$ representations. The notation is such that $(2, 1)$ changes sign under one Z_2 , but not the other.

Let us now count the massless states that survive this compactification. There is one untwisted sector, as usual. In the twisted sectors, there are massless states located at the fixed-point sets of the twisting elements. This means that there are 16 fixed four-tori associated with a twist by one Z_2 , and 16 fixed four-tori associated with the other. There are also 256 fixed points associated with a simultaneous twist by both Z_2 's.

Consider first the untwisted sector. The up light-cone has $(X^i, \tilde{S}^a, S^a) = (8_V, 8_L, 8_L)$. Since the boundary conditions are not twisted, the vacuum energy is

TABLE 5
 $Z_2 \otimes Z_2$ orbifold: The untwisted, massless, up-moving physical spectrum

$ \tilde{i}\rangle \otimes i\rangle$	$ \tilde{i}\rangle \otimes \dot{a}\rangle$	$ \dot{\tilde{a}}\rangle \otimes i\rangle$	$ \dot{\tilde{a}}\rangle \otimes \dot{a}\rangle$
$4(2, 1) \times (2, 1)$			$16(1, 1) \times (1, 1)$
$4(1, 2) \times (1, 2)$			$(2, 2) \times (2, 2)$

zero and the fermions have zero modes. Because of the zero modes, the left- and right-moving vacua fill out the $(8_V, 8_R)$ representation of the $O(8)$ Clifford algebra. The physical states are obtained by tensoring left- and right-moving states, and picking out those invariant under $Z_2 \otimes Z_2$. The results are collected in table 5; we see that there are a total of 64 bosons and no fermions. For the down light-cone, the vacua fill out the $(8_V, 8_L)$ representation of the Clifford algebra. This gives rise to a different enumeration of the physical states. As shown in table 6, the down-moving spectrum contains 64 bosons *plus* 64 fermions.

Now let us consider the sector twisted by both Z_2 elements. For the up light-cone, the fields are $(8_V, 8_L)$, so both the fermions and the bosons obey antiperiodic boundary conditions. The vacuum energy is zero, but there are no fermion zero modes, so there is one bosonic state. Actually, since there are 256 fixed points, this sector contributes 256 bosons to the up-moving spectrum. For the down light-cone, the fields are $(8_V, 8_R)$, and one readily sees that the bosons change sign while the fermions do not. Thus the twisted boundary conditions are antiperiodic for the bosons and periodic for the fermions. The vacuum energy is $+\frac{1}{2}$, and there are no massless states.

The two singly-twisted sectors may be treated similarly. We omit the details, and give the complete census of massless states in table 7. We see that there is a net asymmetry of 256 up-moving bosons and 160 down-moving complex fermions. Let us compare this spectrum with that expected from compactification of chiral type II supergravity on $K3 \otimes K3$.

Recall that chiral type II supergravity contains three potentially anomalous ten-dimensional fields: one positive chirality complex spin- $\frac{3}{2}$ field ψ_m , one negative chirality complex spin- $\frac{1}{2}$ field χ , and one real self-dual antisymmetric tensor field F . The gravitational anomalies of these fields cancel, so type II supergravity is anomaly-free [6].

TABLE 6
 $Z_2 \otimes Z_2$ orbifold: The untwisted, massless, down-moving physical spectrum

$ \tilde{i}\rangle \otimes i\rangle$	$ \tilde{i}\rangle \otimes \dot{a}\rangle$	$ \dot{\tilde{a}}\rangle \otimes i\rangle$	$ \dot{\tilde{a}}\rangle \otimes \dot{a}\rangle$
$4(2, 1) \times (2, 1)$	$4(2, 1) \times (2, 1)$	$4(2, 1) \times (2, 1)$	$4(2, 1) \times (2, 1)$
$4(1, 2) \times (1, 2)$	$4(1, 2) \times (1, 2)$	$4(1, 2) \times (1, 2)$	$4(1, 2) \times (1, 2)$

TABLE 7
 $Z_2 \otimes Z_2$ orbifold: The complete census of massless states

	$1 \otimes 1$	$1 \otimes Z_2$	$Z_2 \otimes 1$	$Z_2 \otimes Z_2$
boson (up)	64	128	128	256
fermion (up)	0	0	0	0
boson (down)	64	128	128	0
fermion (down)	64	128	128	0

How do these fields compactify on $K3 \otimes K3$? The 256 up-moving bosons of table 7 are equivalent to 256 self-dual tensor fields. They can be identified with the zero modes of the ten-dimensional antisymmetric tensor field F . The equations of motion for F are just the self-duality equation $F = *F$, plus the Bianchi identity $dF = 0$. Self-dual harmonic two-forms on $K3 \otimes K3$ give rise to self-dual antisymmetric tensor fields in two dimensions. Since the signature τ is the difference between the number of self-dual and anti-self-dual harmonic two-forms, there is a net asymmetry of 256 self-dual tensors in two dimensions. Here we have used the fact that $\tau(K3 \otimes K3) = \tau(K3) \cdot \tau(K3) = 256$.

The negative chirality spin- $\frac{1}{2}$ field in ten dimensions gives rise to four negative chirality two-dimensional spin- $\frac{1}{2}$ fields. This can be seen by splitting the ten-dimensional Dirac equation into external and internal parts,

$$\not{D}^{(\text{ext})}\chi + \not{D}^{(\text{int})}\chi = 0. \quad (17)$$

The zero modes of $\not{D}^{(\text{int})}$ correspond to massless spinors in two dimensions. The net asymmetry is given by the index of the Dirac operator on $K3 \otimes K3$:

$$\text{ind}_D(K3 \otimes K3) = \text{ind}_D(K3) \cdot \text{ind}_D(K3) = (-2)^2 = 4. \quad (18)$$

The reduction of the spin- $\frac{3}{2}$ field is a little more complicated. It splits into four up-moving spin- $\frac{3}{2}$ fields and 156 down-moving spin- $\frac{1}{2}$ fields. This can be seen by writing the Rarita-Schwinger equation in the gauge $\gamma \cdot \psi = 0$,

$$\not{D}^{(\text{ext})}\psi_m + \not{D}^{(\text{int})}\psi_m = 0. \quad (19)$$

For $m = 1$ and 2 , $\not{D}^{(\text{int})}$ is the Dirac operator on $K3 \otimes K3$, and the zero modes give four spin- $\frac{3}{2}$ fields in two dimensions. For $m = 3, \dots, 10$, $\not{D}^{(\text{int})}$ is the Rarita-Schwinger operator on one $K3$, and the Dirac operator on the other. The 156 zero modes correspond to spin- $\frac{1}{2}$ fields in two dimensions. The net asymmetry is given by the Rarita-Schwinger index [7] on $K3 \otimes K3$,

$$\begin{aligned} \text{ind}_{\text{RS}}(K3 \otimes K3) &= 2 \text{ind}_D(K3) \cdot \text{ind}_{\text{RS}}(K3) \\ &+ \text{ind}_D(K3) \cdot \text{ind}_D(K3) = -156, \end{aligned} \quad (20)$$

where we have used the fact^{*} that $\text{ind}_{\text{RS}}(\text{K3}) = 40$. The last term subtracts the ghost contribution.

Thus we have seen that the compactification of ten-dimensional supergravity on $\text{K3} \otimes \text{K3}$ yields 256 antisymmetric tensors, 160 spin- $\frac{1}{2}$ fermions, and four spin- $\frac{3}{2}$ gravitini. Since in two dimensions spin- $\frac{3}{2}$ fields are pure gauge, the counting of physical degrees of freedom in the orbifold analysis agrees with that resulting from index theorems in the field theory limit.

It is interesting to note that the two-dimensional gravitini and their ghosts, which result from compactification of the ten-dimensional gravitino and its ghosts, give an effective contribution to the gravitational anomaly of -24 times that of a spin- $\frac{1}{2}$ field. This differs from the value of -23 that is obtained by quantizing a two-dimensional gravitino with three spinor ghosts, as would be done by following the correct quantization procedure in higher dimensions.

3.4. COSMIC ORBIFOLD

Finally we come to the Z_6 orbifold. As indicated earlier, this is a particularly interesting case since it can be thought of as a cosmic string embedded in the Z_3 orbifold vacuum discussed in ref. [2]. We now take the underlying string theory to be heterotic since the vacuum (far from the cosmic string) has a fairly realistic phenomenology. The spectrum calculations are rather involved, so we will only summarize our results and point out the nontrivial technical issues that arise.

Because the theory is heterotic, the right- and left-movers must be treated separately. The right-movers are supersymmetric and can be described by a Green-Schwarz theory, just as in the above examples. As before, we need the transformation properties of $O(8)$ vectors and spinors under the holonomy group. The Z_6 holonomy element g lies in $SU(4)$, and its square belongs to the center of $SU(3)$. The obvious decomposition chain is

$$O(8) \rightarrow SU(4) \times U(1)_b \rightarrow SU(3) \times U(1)_a \times U(1)_b. \quad (21)$$

Under the first step of this chain, we have

$$\begin{aligned} 8_V &\rightarrow 4(1) + \bar{4}(-1), \\ 8_L &\rightarrow 4(-1) + \bar{4}(1), \\ 8_R &\rightarrow 6(0) + 1(2) + 1(-2), \end{aligned} \quad (22)$$

^{*} In obtaining ind_{RS} it is important to subtract the ghost contributions correctly. In the standard quantization of a spin- $\frac{3}{2}$ field, there are two spin- $\frac{1}{2}$ ghosts of the same chirality as ψ_m , associated with the constraint $\partial \cdot \psi = 0$ and with invariance under $\delta \psi_m = \partial_m \epsilon$, and one spin- $\frac{1}{2}$ ghost of the opposite chirality, corresponding to the constraint $\gamma \cdot \psi = 0$. This accounts for the difference between the above expression for $\text{ind}_{\text{RS}}(\text{K3})$ and that in ref. [8], where only two ghosts of the same chirality were subtracted.

TABLE 8
Cosmic orbifold: Phases associated with the right-movers

O(8) rep	SU(3) rep	Helicity	U(1) _a phase
8 _V	3	0	$e^{i\pi/3}$
	$\bar{3}$	0	$e^{-i\pi/3}$
	1	1	-1
	1	-1	-1
8 _L	3	$\frac{1}{2}$	$e^{i\pi/3}$
	$\bar{3}$	$-\frac{1}{2}$	$e^{-i\pi/3}$
	1	$\frac{1}{2}$	-1
	1	$-\frac{1}{2}$	-1
8 _R	3	$-\frac{1}{2}$	$e^{2i\pi/3}$
	$\bar{3}$	$\frac{1}{2}$	$e^{2i\pi/3}$
	1	$-\frac{1}{2}$	1
	1	$\frac{1}{2}$	1

which under the second step reduces to

$$\begin{aligned}
 8_V &\rightarrow 3(-1, 1) + 1(3, 1) + \bar{3}(1, -1) + 1(-3, -1), \\
 8_L &\rightarrow 3(-1, -1) + 1(3, -1) + \bar{3}(1, 1) + 1(-3, 1), \\
 8_R &\rightarrow 3(2, 0) + 1(0, 2) + \bar{3}(-2, 0) + 1(0, -2).
 \end{aligned} \tag{23}$$

In both of these cases, the numbers in parentheses are the relevant U(1) charges. One easily sees that our Z_6 element g multiplies states in the above decompositions by the phase $e^{-i\pi a/3}$. This phase determines the spectrum and the twisted sectors; all physical states must have overall phase unity. We are also interested in the U(1)_b quantum number because rotations about the z -axis transform the states by $e^{-i\pi(a+b)/4}$. This phase plays the role of helicity, and helps us identify states as bosons or fermions. These transformation properties are summarized in table 8.

There are several ways to treat the left-movers. It is convenient to regard the left-movers as an 8_V of bosons (which transform under the holonomy as described above) plus 32 NSR fermions which linearly realize an $O(16) \times O(16)'$ subgroup of the full $E_8 \times E_8'$ gauge group. By assigning a group action on the gauge coordinates, consistent with modular invariance, the orbifold breaks gauge as well as spacetime symmetries. We choose the group action to be generated by the same Z_6 element as above, but located in the SU(4) that arises by breaking SO(16) to $SO(10) \times SU(4)$. Since our Z_6 element g commutes with SU(3), the unbroken gauge group is $SO(10) \times SU(3) \times E_8'$, and the states of the theory are classified accordingly.

TABLE 9
Cosmic orbifold: The up-moving, massless physical spectrum

Sector	Number	Spin	$O(10) \times SU(3) \times E'_8$	L_z
untwisted	10	$(0, \frac{1}{2})$	(1, 1, 1)	(even, odd)
	1	$(\frac{3}{2}, 2)$	(1, 1, 1)	(odd, even)
	2	$(\frac{1}{2}, 1)$	(16, 1, 1)	(odd, even)
	3	$(0, \frac{1}{2})$	(16, 3, 1)	(even, odd)
	1	$(\frac{1}{2}, 1)$	(1, 1, 248)	(even, odd)
	1	$(\frac{1}{2}, 1)$	(45, 1, 1)	(even, odd)
	1	$(\frac{1}{2}, 1)$	(1, 8, 1)	(even, odd)
	1	$(\frac{1}{2}, 1)$	(1, 1, 1)	(even, odd)
	3	$(0, \frac{1}{2})$	(10, 3, 1)	(odd, even)
	3	$(0, \frac{1}{2})$	(1, 3, 1)	(odd, even)
twisted g, g^5				
twisted g^2, g^4	27	$(0, \frac{1}{2})$	(16, 1, 1)	(even, odd)
	81	$(0, \frac{1}{2})$	(1, $\bar{3}$, 1)	(even, odd)
	27	$(0, \frac{1}{2})$	(10, 1, 1)	(odd, even)
	27	$(0, \frac{1}{2})$	(1, 1, 1)	(odd, even)
twisted g^3	1	F	(10, 1, 1)	

In working out the spectrum of the NSR fermions, it is necessary to project not only onto group invariant states, but also onto states with the correct world-sheet fermion number. The results of a calculation of the spectrum of massless physical states are presented in tables 9 and 10. Table 9 refers to the up light-cone, and table 10 to the down. In the first column the twisted sectors are labelled by their associated holonomy element (g is the generator of the Z_6 holonomy group). The states are organized into representations of $O(10) \times SU(3)$, and their spins and multiplicities are indicated as well.

What do we learn from this spectrum? Strings in sectors twisted an odd number of times have no coordinate zero modes. They are effectively bound to the cosmic string and behave like genuine two-dimensional objects. For that reason we do not give their spin, but just indicate whether they are bosons or fermions. On the other hand, strings in the sectors twisted an even number of times *do* have coordinate zero modes in the two spatial directions transverse to the cosmic string. States in these sectors behave like four-dimensional massless particles. They are not bound to the string, so we also indicate their spin. It is important to remember that the coordinate zero-mode wave functions transform under the holonomy group. This implies that there are different sets of states associated with even and odd angular momenta about the z -axis. This is indicated by the last column in the tables. In a compactification down to two dimensions, where the dimensions transverse to the

TABLE 10
Cosmic string: The down-moving, massless physical spectrum

Sector	Number	Spin	$O(10) \times SU(3) \times E'_8$	L_Z
untwisted	10	$(0, \frac{1}{2})$	$(1, 1, 1)$	(even, even)
	1	$(\frac{3}{2}, 2)$	$(1, 1, 1)$	(even, even)
	2	$(\frac{1}{2}, 1)$	$(16, 1, 1)$	(even, even)
	3	$(0, \frac{1}{2})$	$(16, 3, 1)$	(even, even)
	1	$(\frac{1}{2}, 1)$	$(1, 1, 248)$	(odd, odd)
	1	$(\frac{1}{2}, 1)$	$(45, 1, 1)$	(odd, odd)
	1	$(\frac{1}{2}, 1)$	$(1, 8, 1)$	(odd, odd)
	1	$(\frac{1}{2}, 1)$	$(1, 1, 1)$	(odd, odd)
	3	$(0, \frac{1}{2})$	$(10, 3, 1)$	(odd, odd)
	3	$(0, \frac{1}{2})$	$(1, 3, 1)$	(odd, odd)
	1	(B, F)	$(16, 1, 1)$	
	3	(B, F)	$(1, 3, 1)$	
twisted g, g^5	12	(B, F)	$(1, 1, 1)$	
	27	$(0, \frac{1}{2})$	$(16, 1, 1)$	(even, even)
	81	$(0, \frac{1}{2})$	$(1, \bar{3}, 1)$	(even, even)
twisted g^2, g^4	27	$(0, \frac{1}{2})$	$(10, 1, 1)$	(odd, odd)
	27	$(0, \frac{1}{2})$	$(1, 1, 1)$	(odd, odd)
	27	$(0, \frac{1}{2})$		
twisted g^3				

string are “small,” the states of nonzero angular momentum are viewed as having finite mass, and are not included in the massless spectrum. In a cosmic string interpretation, where two of the transverse dimensions are “large,” all angular-momentum states are treated on the same footing.

We have classified the states according to their $O(10) \times SU(3)$ representations. On the other hand, we have argued that, far from the string, the vacuum should be identified with that of the Z_3 orbifold, with gauge group $E_6 \times SU(3)$. Thus the states that exist far from the string, i.e. those in the even twisted sectors, should actually fall into $E_6 \times SU(3)$ representations. In fact, if we ignore the distinction between even and odd orbital angular momenta, as is appropriate for states far from the string, we see that the even-twist states do fall into $E_6 \times SU(3)$ representations. The states are precisely those of the Z_3 orbifold. (For instance, the $(0, \frac{1}{2})$ states in the twice-twisted sector can be assembled into 27 $(27, 1)$ ’s of $E_6 \times SU(3)$.) We regard this spectrum as an interesting, almost realistic example of a cosmic string that can be built in string theory. It is very different from the type of string expected in grand unification models.

It is not hard to check that the cosmic string spectrum is anomalous. Since the field theory limit of the heterotic string is anomaly-free only after the addition of Green-Schwarz counterterms, we expect the same to hold here as well. To illustrate

this, consider the gravitational anomaly in two dimensions. It can be derived from the chiral anomaly in four dimensions using the descent equation [9]

$$\mathrm{tr} R^2 = dQ_{3L},$$

$$\delta_L Q_{3L} = dQ_{2L}^1, \quad (24)$$

where δ_L indicates the variation with respect to an infinitesimal Lorentz transformation. The anomaly is proportional to Q_{2L}^1 and may be cancelled by adding a term $\int B$ to the action, where B is an antisymmetric tensor field with variation $\delta_L B = -Q_{2L}^1$. In two dimensions, the field strength $H = dB$ vanishes, so B is nonpropagating. The physical interpretation of this method of anomaly cancellation is obscure. Certainly a full string-theoretic calculation must lead to a consistent anomaly-free theory; it is just the low-energy field-theoretic description that appears peculiar.

4. Discussion

The orbifold compactifications we have constructed resemble cosmic strings in that the four-dimensional spacetime geometry is conical. These compactifications are classical solutions since they correspond to conformally invariant string theories. In four-dimensional Calabi-Yau compactifications (and in their orbifold counterparts), string loop corrections are not expected to lead to instabilities because of the unbroken supersymmetry. For our compactifications, the situation is less clear. There is no topology that guarantees the stability of the solutions, as there is for ordinary cosmic strings. Furthermore, there are no fermion zero-modes in the up-moving sigma-model, and as a result we found a nonvanishing contribution to the vacuum energy from this sector. As discussed before, this term is properly interpreted as a correction to the tension μ of the cosmic string. Since this correction is proportional to the difference between the number of massless bosons and fermions, it can be large even when the string coupling is small. This correction acts as a line source for the dilaton field, resulting in dilaton emission.

What then is the final fate of our solution when string loop corrections are included? We can imagine two possibilities. One is that the configuration decays by dilaton emission to a configuration with no deficit angle. The other possibility is that there might be a solution to the string equations of motion with a renormalized but nonzero deficit angle, and a spatially varying dilaton field.

It would be very interesting to find such a solution. This situation has ingredients that are reminiscent of some of the major unsolved problems in string theory. There has been little progress in understanding the mechanism for supersymmetry breaking in string theory, or in understanding how the cosmological constant and dilaton vacuum expectation value are determined once supersymmetry is broken. In our

orbifold compactifications, supersymmetry is broken in what seems to be the most innocuous way possible since only the massless modes are not supersymmetric. Analyzing radiative corrections and their effect on the dilaton field should be much simpler here than in a string theory where supersymmetry breaking affects all string modes. In addition to providing a useful laboratory for addressing these purely string-theoretic questions, it is possible that the renormalized values of the string energy density and deficit angle might be such that these strings are of cosmological interest.

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Appendix

In this appendix we compute the spectrum of the Z_2 orbifold using the Neveu-Schwarz-Ramond formalism. In the light-cone gauge, this means that there are eight bosonic coordinates X^i and eight Majorana fermions ψ^i . The bosons and the fermions transform as vectors of $O(8)$, so world-sheet supersymmetry is manifest.

The supersymmetric sigma model built from the X^i and the ψ^i is well-defined for any choice of spin structure on the world sheet. However, modular invariance of the string requires that one sum over spin structures and project onto physical states with a Gliozzi-Scherk-Olive projection operator. In the Neveu-Schwarz sector (with antiperiodic boundary conditions on the ψ^i), the GSO operator is given by $\frac{1}{2}(1 - (-)^F)$. This choice eliminates the tachyon from the physical spectrum. In the Ramond sector (with periodic boundary conditions), the GSO projector is $\frac{1}{2}(1 \pm (-)^F)$. In flat space – or on a manifold of $SU(3)$ holonomy – the choice of sign is irrelevant. For the case considered here, we shall see that there is physics in the choice.

Let us compute the spectrum of the Z_2 orbifold. We start with the untwisted Neveu-Schwarz sector, with eight periodic bosons X^i and eight antiperiodic fermions ψ^i . The bosons and the fermions have left- and right-moving creation and annihilation operators. The bosonic operators are moded with integers, while the fermions are moded with half-odd-integers. Independent GSO projections are performed on the left- and right-moving sectors. Let us focus for the moment on the right-movers. The right-moving vacuum is at mass level $-\frac{1}{2}$, but it is annihilated by the right-moving GSO projection. The first physical state is at mass level zero. It is a world-sheet fermion and a spacetime vector: $\psi_{-1/2}^i|0\rangle$. Higher states are constructed in a similar fashion. The physical spectrum, for both left- and right-movers, is shown in table 11.

In the untwisted Ramond sector, the bosons and fermions are both integer moded. Since there are eight fermions and eight bosons, the vacuum is at mass level zero. However, there are eight fermionic zero modes, so the vacuum is degenerate. To see this, let us restrict our attention to the right-movers, and collect the eight

TABLE 11
 Z_2 orbifold: The untwisted spectrum in the Neveu-Schwarz sector

Mass	Left-movers		Right-movers	
	parity +	parity -	parity +	parity -
0		$\tilde{\psi}_{-1/2}^j 0\rangle$		$\psi_{-1/2}^j 0\rangle$
1	$\tilde{\alpha}_{-1}^j \tilde{\psi}_{-1/2}^j 0\rangle$	$\tilde{\psi}_{-3/2}^j 0\rangle$ $\tilde{\psi}_{-1/2}^i \tilde{\psi}_{-1/2}^j \tilde{\psi}_{-1/2}^k 0\rangle$	$\alpha_{-1}^j \psi_{-1/2}^j 0\rangle$	$\psi_{-3/2}^j 0\rangle$ $\psi_{-1/2}^i \psi_{-1/2}^j \psi_{-1/2}^k 0\rangle$

zero modes into four complex modes ψ_0^i , and four conjugates $\bar{\psi}_0^{\bar{j}}$. The zero modes obey the quantization conditions

$$\{\psi_0^i, \psi_0^j\} = \{\bar{\psi}_0^{\bar{i}}, \bar{\psi}_0^{\bar{j}}\} = 0, \quad \{\psi_0^i, \bar{\psi}_0^{\bar{j}}\} = g^{ij}. \quad (\text{A.1})$$

We define the Clifford vacuum $|0\rangle$ to be annihilated by the $\bar{\psi}_0^{\bar{j}}$,

$$\bar{\psi}_0^{\bar{j}} |0\rangle = 0. \quad (\text{A.2})$$

The zero-energy states are built on the Clifford vacuum with the creation operators ψ_0^i :

$$\begin{aligned} &|0\rangle, \\ &\psi_0^i |0\rangle, \\ &\psi_0^i \psi_0^j |0\rangle, \\ &\psi_0^i \psi_0^j \psi_0^k |0\rangle, \\ &\psi_0^i \psi_0^j \psi_0^k \psi_0^l |0\rangle. \end{aligned} \quad (\text{A.3})$$

We see that there are a total of 16 states, which split naturally into eight states of even fermion number, and eight of odd. In terms of $SU(4)$ representations, the eight even states fill out a $6 + 1 + 1$, while the eight odd states make up a $4 + \bar{4}$. These are just the $SU(4)$ decompositions of the two $O(8)$ spinors, as shown in eq. (5). The states of even fermion number form an 8_R of $O(8)$ which we denote by $|\dot{a}\rangle$. The states of odd fermion number make up an 8_L which we denote by $|a\rangle$.

The physics of the two Ramond-sector GSO projections is now clear. For compactifications to two dimensions, the projector $\frac{1}{2}(1 + (-)^F)$ describes the up-moving states, while $\frac{1}{2}(1 - (-)^F)$ describes the down. In each case, the massive states are built with the integer-moded bosonic and fermionic creation operators. The physical up- and down-moving states are collected in tables 12 and 13. (By way of contrast, note that the states in the Neveu-Schwarz sector move both up and down.)

TABLE 12
Z₂ orbifold: The up-moving, untwisted spectrum in the Ramond sector

Mass	Left-movers		Right-movers	
	parity +	parity -	parity +	parity -
0	$ \dot{\tilde{a}}\rangle$		$ \dot{a}\rangle$	
1	$\tilde{\psi}'_{-1} \dot{\tilde{a}}\rangle$	$\tilde{\alpha}'_{-1} \dot{\tilde{a}}\rangle$	$\psi'_{-1} \dot{a}\rangle$	$\alpha'_{-1} \dot{a}\rangle$

TABLE 13
Z₂ orbifold: The down-moving, untwisted spectrum in the Ramond sector

Mass	Left-movers		Right-movers	
	parity +	parity -	parity +	parity -
0		$ \tilde{a}\rangle$		$ a\rangle$
1	$\tilde{\alpha}'_{-1} \tilde{a}\rangle$	$\tilde{\psi}'_{-1} \tilde{a}\rangle$	$\alpha'_{-1} a\rangle$	$\psi'_{-1} a\rangle$

Let us now turn to the twisted sectors, and first compute the Neveu-Schwarz spectrum. The twisted Neveu-Schwarz sector has integer-moded fermions and half-odd-integer bosons. The vacuum is at mass level $+\frac{1}{2}$. Since there are eight fermion zero modes, the vacuum is 16-fold degenerate. As before, it splits into an 8_L and an 8_R of $O(8)$. The 8_R has even fermion number, so it is annihilated by the Neveu-Schwarz GSO projection. (The twisted GSO projector is identical to the untwisted projector since the net fermion number of the Clifford vacuum does not change under a Z_2 twist of all eight fermions.) The complete spectrum is given in table 14.

Finally, we treat the twisted Ramond sector, where the fermions and the bosons are both half-odd-integer moded. The vacuum energy is zero, and since there are no fermion zero modes, the vacuum is unique. The Ramond GSO projection keeps this vacuum in the up-moving spectrum, but annihilates it in the down. The massive states are built as usual, and the physical up- and down-moving spectra are listed in tables 15 and 16.

TABLE 14
Z₂ orbifold: The twisted spectrum in the Neveu-Schwarz sector

Mass	Left-movers		Right-movers	
	parity +	parity -	parity +	parity -
$\frac{1}{2}$		$ \tilde{a}\rangle$		$ a\rangle$
1	$\tilde{\alpha}'_{-1/2} \tilde{a}\rangle$		$\alpha'_{-1/2} a\rangle$	

TABLE 15
 Z_2 orbifold: The up-moving, twisted spectrum in the Ramond sector

Mass	Left-movers		Right-movers	
	parity +	parity -	parity +	parity -
0	$ 0\rangle$		$ 0\rangle$	
$\frac{1}{2}$		$\tilde{\alpha}_{-1/2}^i 0\rangle$		$\alpha_{-1/2}^i 0\rangle$
1	$\tilde{\alpha}_{-1/2}^i \tilde{\alpha}_{-1/2}^j 0\rangle$ $\tilde{\psi}_{-1/2}^i \tilde{\psi}_{-1/2}^j 0\rangle$		$\alpha_{-1/2}^i \alpha_{-1/2}^j 0\rangle$ $\psi_{-1/2}^i \psi_{-1/2}^j 0\rangle$	

TABLE 16
 Z_2 orbifold: The down-moving, twisted spectrum in the Ramond sector

Mass	Left-movers		Right-movers	
	parity +	parity -	parity +	parity -
$\frac{1}{2}$		$\tilde{\psi}_{-1/2}^i 0\rangle$		$\psi_{-1/2}^i 0\rangle$
1	$\tilde{\alpha}_{-1/2}^i \tilde{\psi}_{-1/2}^j 0\rangle$		$\alpha_{-1/2}^i \psi_{-1/2}^j 0\rangle$	

Tables 11–16 contain the full spectrum of the Z_2 orbifold, computed with the Neveu-Schwarz-Ramond formalism. Let us now compare it with the spectrum derived from the Green-Schwarz formalism, which is collected in tables 1–4. For simplicity, let us restrict our attention to the untwisted, up-moving sector – the others may be treated in a similar fashion. The untwisted Neveu-Schwarz states are given in table 11. They are all spacetime bosons. At the massless level, there are eight bosons of parity +. At mass level 1, there are 64 bosons of parity +, and 64 bosons of parity -. The corresponding Ramond states are listed in table 12. These states are all spacetime fermions. At the massless level, there are eight fermions of parity +, while at mass level 1, there are 64 fermions of parity + and 64 of parity -.

The up-moving spectrum computed with the Green-Schwarz formalism is shown in table 1. Counting bosons and fermions, we see that it agrees with the Neveu-Schwarz-Ramond spectrum, mass level by mass level. Note that one should not be alarmed by the different $O(8)$ representations that occur in the two formalisms, for the various $O(8)$ representations just count bosons and fermions from the two-dimensional point of view.

It is easy to check that the GS-NSR equivalence holds for both up- and down-movers, in the twisted and untwisted sectors. The argument can be extended to all mass levels by computing the various partition functions. We have done this, and we have found that the GS and NSR partition functions are identical in each sector of Hilbert space.

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