

INSTANTONS AND (SUPER-) SYMMETRY BREAKING IN (2+1) DIMENSIONS*

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The connection between instantons and the breaking of supersymmetry and ordinary symmetries is studied in a variety of (2+1)-dimensional gauge theories.

1. Introduction

Unbroken supersymmetry requires degeneracy between Bose and Fermi degrees of freedom [1]. Since no such degeneracy is actually observed, any supersymmetric model of nature must include a mechanism for supersymmetry breaking [2]. The scale of this breaking may be of order the Planck mass – in which case only local supersymmetry (supergravity) is of relevance to the real world. If this is the case, then supersymmetry is relevant to particle physics at ordinary energies only in a very indirect fashion. In particular, it seems unclear how supersymmetry broken at the Planck mass could provide an explanation for why the mass scales of ordinary physics are so much smaller than the Planck mass.

On the other hand, if supersymmetry is broken at energies of say a few TeV, it is quite plausible that this breaking could trigger the usual $SU(2)_L$ breaking if Higgs fields, which were forced to be massless by supersymmetry, acquired vacuum expectation values once supersymmetry was broken. However, one must still explain why the scale of supersymmetry breaking is so much less than the Planck mass.

One attractive possibility would be that supersymmetry is unbroken in perturbation theory, but is “dynamically broken” through small non-perturbative effects [3]. For this to happen, in a weakly coupled theory, a fermion must exist which is massless to all orders in perturbation theory but has no coupling to the supersymmetry current at zero momentum. If instantons induce such a coupling, this fermion becomes the Goldstino of spontaneously broken supersymmetry. Writing the zero-momentum coupling as

$$\langle 0 | S_{\mu\alpha} | \psi_\beta \rangle = f(\gamma_\mu)_{\alpha\beta}, \quad (1.1)$$

the vacuum energy is $E_0 = f^2$. Unfortunately, it is difficult to know whether or not this occurs in realistic models in 3+1 dimensions. There are general arguments

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that indicate that dynamical supersymmetry breaking does *not* occur in a wide class of models [5]. However, these arguments do not apply directly to the realistic case of models with complex fermion representations. On the positive side, it *is* known that instantons can induce supersymmetry breaking in models in $0+1$ dimensions (supersymmetric quantum mechanics) and it has been suggested that this may also be the case in $2+1$ dimensions [3].

The main purpose of this paper is to examine the role of instantons more carefully in $(2+1)$ -dimensional supersymmetry. The first problem we confront is that gauge-invariant mass terms [6] for gauge fields exist in $(2+1)$ dimensions which prevent the occurrence of either supersymmetry breaking or instantons. We must forbid these terms by imposing discrete symmetries since there do not appear to be non-renormalization theorems [7] in $(2+1)$ -dimensional supersymmetry that could prevent generation of a photon mass in perturbation theory. [There are non-renormalization theorems in $N=2$ supersymmetry.] This is discussed in the appendix. In sect. 2 we begin our discussion of instanton effects by considering a simple non-supersymmetric Higgs model with fermions. As well as introducing many of the techniques needed later, this model has some interesting features of its own: a fermion number symmetry is spontaneously broken, the photon is the Goldstone boson, and all symmetry breaking amplitudes are exponentially small. In sect. 3 we consider supersymmetric Higgs theories in which only the photon and photino are massless in perturbation theory. We will find that instanton effects, rather than breaking supersymmetry, give the photon and photino an exponentially small mass. In sect. 4 we consider more general models with matter fields whose expectation values are undetermined in perturbation theory. We find that, in general, instanton effects determine expectation values for these fields without breaking supersymmetry. [There is a possibility that in some theories no vacuum state exists and the effective potential is exponentially small but positive except at infinite field strength.] In sect. 5 we consider a model with $N=2$ supersymmetry which is a special case of the models in sect. 4 and sect. 2. It can be obtained by dimensional reduction from the minimal $N=1$ supersymmetric gauge theory in $(3+1)$ dimensions. We find that instanton effects violate a non-renormalization theorem of perturbation theory but apparently do not break supersymmetry in the usual sense. In sect. 6 we show that our results are compatible with those that could be obtained from the general $\text{tr}(-1)^F$ arguments of ref. [5]. Sect. 7 contains conclusions. In the appendix we establish our notation and describe C , P and T symmetries in $2+1$ dimensions.

2. Instantons and symmetry breaking

Compact electrodynamics in $2+1$ dimensions was originally studied by Polyakov, who showed in a beautiful paper [8] that as a result of instanton effects, the photon acquires a mass.

In this paper, we will study compact electrodynamics in $2+1$ dimensions in the presence of fermions. Although our main interest will be to examine supersymmetric theories, it will be useful in this section to examine some phenomena that result from the coupling to fermions of compact electrodynamics but do not depend on supersymmetry.

We will study theories in which some compact gauge group, which for simplicity we will take to be $SU(2)$, is spontaneously broken down to $U(1)$. We wish to study theories in which the photon is massless in perturbation theory because only then are there instantons. For reasons explained in the appendix, in $2+1$ dimensions the photon is massless in perturbation theory only if there is an unbroken P or CP symmetry, so we will consider only theories in which in perturbation theory there is such a symmetry.

We wish to ask whether instantons trigger the spontaneous breaking of supersymmetry, of CP , or of various fermion number conservation laws which may be present. We also wish to determine under what conditions the photon acquires a mass from instanton effects.

We will consider in this section an $SU(2)$ gauge theory coupled to a scalar field Φ in the adjoint representation of $SU(2)$ and to a complex Fermi field ψ also in the adjoint representation. The euclidean space lagrangian density is

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(D_\mu\Phi)^2 + V(\Phi) + \bar{\psi} \cdot (i\not{D} + g\Phi \times)\psi, \quad (2.1)$$

which, in the absence of the fermions, would be the model considered by Polyakov.

This model possesses a CP symmetry

$$\Phi(x) \rightarrow -\Phi(-x), \quad A_\mu(x) \rightarrow -A_\mu(-x), \quad \psi(x) \rightarrow i\psi(-x), \quad \bar{\psi}(x) \rightarrow i\bar{\psi}(-x). \quad (2.2)$$

Note that – in euclidean space – $\bar{\psi}$ does *not* transform as the conjugate of ψ . It must be regarded as an independent Grassman variable. (In particular, its euclidean rotation matrix cannot be obtained from conjugating ψ .)

This model also possesses the continuous fermion number conservation law

$$\psi \rightarrow e^{i\theta}\psi, \quad \bar{\psi} \rightarrow e^{-i\theta}\bar{\psi}. \quad (2.3)$$

Anticipating ourselves a bit, we will find that, because of fermion zero modes, the instanton amplitudes do not respect the conservation law (2.3). However, this has nothing to do with anomalies. Indeed, the current corresponding to (3) is

$$J_\mu = \bar{\psi}\gamma_\mu \cdot \psi. \quad (2.4)$$

It is completely conserved and anomaly free. Rather, the violation of fermion number conservation that we will find in instanton amplitudes is a spontaneous breakdown of the symmetry. As always, spontaneous symmetry breaking requires the existence of a Goldstone boson, and in this case it turns out that the relevant Goldstone boson is the photon. This curious situation is possible because in $2+1$



Fig. 1. One-loop Feynman diagram contributing to the amplitude for the fermion number current J_μ to create a photon of momentum P and polarization ϵ^λ from the vacuum.

dimensions there is no such thing as spin for massless particles, so that any massless boson – such as the photon – can be a Goldstone boson.

Actually, to see that the photon is a Goldstone boson in the model considered here does not require an instanton calculation. The relevant question is whether there is a non-zero matrix element $\langle 0 | J_\mu | \gamma \rangle$ for the current J_μ to create a one-photon state from the vacuum. Although this matrix element is zero at the tree level, at the one-loop level one encounters the Feynman diagram of fig. 1 which is definitely non-zero and for an on-shell photon ($p^2 = 0$) gives

$$\langle 0 | J_\mu | \gamma \rangle = \frac{e}{2\pi} \epsilon_{\mu\nu\gamma} p^\nu \epsilon^\gamma, \quad (2.5)$$

where p^ν and ϵ^γ are the photon momentum and polarization vector, respectively. The non-zero value means that the photon is a Goldstone boson.

Eq. (2.5) has a peculiar appearance because – although the photon in 2+1 dimensions is physically equivalent to a massless scalar – the kinematical description used here (with the photon polarization vector ϵ^γ , which in 2+1 dimensions is unique except for sign and up to a gauge transformation) is not usually used for massless scalars. To obtain a more standard-looking expression, we may consider the current two-point function $\langle 0 | J_\nu(p) J_\mu(-p) | 0 \rangle$ which, at the two-loop level (fig. 2), exhibits the pole characteristic of spontaneous symmetry breaking

$$\langle 0 | J_\mu(p) J_\nu(-p) | 0 \rangle = \left(\frac{e}{2\pi} \right)^2 \frac{p_\mu p_\nu}{p^2} + \text{non-singular}. \quad (2.6)$$

Given that in perturbation theory, the photon is a Goldstone boson, and, therefore, the conserved charge does not annihilate the vacuum, what connection is there between instantons and symmetry breaking? The answer to this is rather surprising. Though in perturbation theory the fermion number symmetry is spontaneously broken, from a formal point of view (because the current creates a massless particle from the vacuum), the symmetry-breaking amplitudes all vanish in perturbation theory. Indeed, according to current algebra, two ingredients are needed to give non-zero symmetry-breaking effects. There must be a Goldstone boson; and it must have non-zero couplings in the zero momentum limit. In this

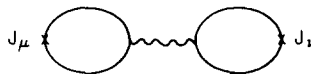


Fig. 2. Two-loop contribution to the fermion number current two-point function.

case, although in perturbation theory the photon is a Goldstone boson, it decouples at zero momentum because of ordinary gauge invariance. The role of instantons is that only at the instanton level does the photon cease to decouple at zero momentum, and only then are there non-zero symmetry breaking effects.

Let us first demonstrate the symmetry-breaking effects of instantons by taking fixed classical boson fields which are CP invariant (such as the monopole) and considering the fermion number and CP symmetries of the fermionic quantum theory. We compute Green functions by functional integration. The vacuum to vacuum amplitude is given by

$$Z = \int (d\bar{\psi})(d\psi) \exp \left(- \int d^3x \bar{\psi} (i\mathcal{D} + g\Phi \times) \psi \right). \quad (2.7)$$

To make this expression meaningful we expand ψ and $\bar{\psi}$ in complete orthonormal sets of functions:

$$\psi(x) = \sum \xi_i \psi_i(x), \quad \bar{\psi}(x) = \sum \bar{\xi}_i \bar{\psi}_i(x), \quad (2.8)$$

where ξ_i and $\bar{\xi}_i$ are (independent) Grassmann variables, and write

$$(d\bar{\psi}) = \prod_i d\bar{\xi}_i, \quad (d\psi) = \prod_i d\xi_i. \quad (2.9)$$

Since $M \equiv (i\mathcal{D} + \Phi \times)$ is not hermitian, we cannot use its eigenfunctions. However it is convenient to use the hermitian operators M^+M and MM^+ :

$$M^+M\psi_i = \lambda_i\psi_i, \quad MM^+\bar{\psi}_i^* = \bar{\lambda}_i\bar{\psi}_i^*. \quad (2.10)$$

Notice that if $\lambda_i \neq 0$ then $\bar{\psi}_i^* = M\psi_i/\sqrt{\lambda_i}$ (and $\bar{\lambda}_i = \lambda_i$). Thus all non-zero eigenvalues are paired. It is the non-pairing of zero modes which leads to symmetry breaking. The number of zero modes of M^+M minus the number of zero modes of MM^+ is the index of M . By the Callias index theorem [9] it equals twice the topological charge. The operators M and M^+ are odd under $x \rightarrow -x$. Therefore ψ_i and $\bar{\psi}_i^*$ are either even or odd. Furthermore for non-zero modes ψ_i and $\bar{\psi}_i^* (=M\psi_i/\sqrt{\lambda_i})$ have opposite parity. If we perform a CP transformation on the fields $\psi, \bar{\psi}$ [eq. (2.2)] this induces a transformation on the Grassmann variables:

$$\begin{aligned} \xi_i &= \int d^3x \bar{\psi}_i^+(x) \cdot \psi(x) \rightarrow i \int d^3x \bar{\psi}_i^+(-x) \cdot \psi(x) = \pm i\xi_i, & (\text{for } \psi_i \text{ even or odd}), \\ \bar{\xi}_j &\rightarrow \pm i\bar{\xi}_j, & (\text{for } \bar{\psi}_j \text{ even or odd}). \end{aligned} \quad (2.11)$$

Thus for all non-zero modes, $d\bar{\xi}_i d\xi_i \rightarrow d\bar{\xi}_i d\xi_i$. The zero modes also come in pairs; however, they have the *same* parity. This follows from T -reversal. If ψ is a zero mode of M^+M (or MM^+) then so is $\gamma_0\psi^*$, which has the same parity. Thus the entire functional integration measure transforms under parity by a factor of $(-1)^\nu$, where ν is the topological charge. Thus, if our background field has (for example)

$\nu = 1$ then only parity odd expectation values are non-zero. In fact only operators with 2 more ψ 's than $\bar{\psi}$'s can be non-zero, in order to "soak up" the zero modes. The analysis of Jackiw and Rebbi [10] shows that, at least for spherically symmetric background fields, M^+M has two zero modes and MM^+ none. Thus

$$\int (d\bar{\psi})(d\psi) e^{-S} \bar{\psi}^T \gamma_0 \psi = 2 \bar{\psi}_1^T \gamma_0 \psi_2 A e^{-S_0}, \quad (2.12)$$

where S_0 is the bosonic action, A contains the various one-loop factors and ψ_1 and ψ_2 are the two zero modes. Thus we see that CP , although a symmetry of S , is not a symmetry of the integration measure. It is explicitly broken by our choice of a non-zero topological charge. A similar analysis can be applied to the fermion number symmetry. We find that under a $U(1)$ transformation by an angle θ the integration measure transforms by $e^{2i\nu\theta}$.

Let us now go back to the full theory with dynamical bosons. When we integrate over all topological charge we expect the symmetries not to be broken explicitly, but perhaps spontaneously. It will be convenient to formulate this issue in terms of an effective lagrangian for the degrees of freedom which are massless to all orders in perturbation theory, namely the photon and the massless components of the fermions, $\Phi \cdot \Psi$, $\Phi \cdot \bar{\Psi}$. These fields are massless to all orders in perturbation theory due to the CP symmetry. The instanton (and anti-instanton) generate mass terms in the effective lagrangian for these $SU(2)$ singlet fermions:

$$\langle \Phi \cdot \Psi_\alpha(x) \Phi \cdot \bar{\Psi}_\beta^T \gamma_0(x) \rangle = A e^{-S_0} 2 \int d^3z \Phi \cdot \Psi_\alpha^{(1)}(x-z) (\Phi \cdot \bar{\Psi}^{T(2)} \gamma_0)_\beta(x-z), \quad (2.13)$$

where $\Psi^{(1)}$ and $\bar{\Psi}^{(2)}$ are the zero-modes and α, β are Dirac indices. Such a contribution to the propagator would arise from a non-local mass term

$$m(x-y) = A e^{-S_0} \vec{\gamma} \int d^3z \Phi \cdot \Psi^{(1)}(x-z) \Phi \cdot \bar{\Psi}^{T(2)} \gamma_0(y-z) \vec{\gamma}. \quad (2.14)$$

In the dilute-gas approximation where classical instanton interactions are ignored and quantum interactions (i.e. corrections to factorization of determinants) are treated in the multiple scattering approximation [11], the multi-instanton contributions to all Green functions simply give multiple insertions of the non-local mass terms. Including the ψ mass term arising from the anti-instanton we obtain an effective action

$$S_{\text{eff}} = \int d^3x \bar{\psi} i \vec{\gamma} \psi + \int d^3x d^3y [\bar{\psi}^T \gamma_0(x) m(x-y) \psi(y) + \bar{\psi}(x) m(x-y) \gamma_0 \bar{\psi}^T(y)]. \quad (2.15)$$

We have represented the massless $SU(2)$ singlet fermions by simply ψ and $\bar{\psi}$. This non-local mass term can be written as a local mass term plus higher derivatives,

the local mass being,

$$m = A e^{-S_0} \left[\int d^3x \not{x} \not{\Phi} \cdot \Psi^{(1)} \right] \left[\int d^3x \Phi \cdot \Psi^{(2)} \gamma_0 \not{x} \right]. \quad (2.16)$$

The equality of m_ψ and $m_{\bar{\psi}}$ follows from C invariance, which maps instanton into anti-instanton. If we broke C by adding a topological term, $(i\alpha/4\pi) \partial_\mu \Phi \cdot \mathbf{B}_\mu / \Phi_0$, to the lagrangian then we would find

$$\mathcal{L}_{\text{eff}} = \bar{\psi} i \partial \psi + \psi^\top \gamma_0 (m e^{i\alpha}) \psi + \bar{\psi} (m e^{-i\alpha}) \gamma_0 \bar{\psi}^\top + [\text{higher derivative terms}]. \quad (2.17)$$

[$m_\psi = m_\psi^*$ follows from CTP .] This effective lagrangian explicitly breaks CP and fermion number in contradistinction to our expectation that these symmetries should be present once all topological sectors are included.

The resolution of this paradox lies in the classical instanton interactions. For instantons or anti-instantons far apart compared to their size ($\sim 1/M_w e^2$) these interactions are coulombic $\sim 1/|x|$. Thus, in the theory without fermions, summing up the multi-instantons corresponds to calculating the Coulomb gas partition function. Following Polyakov [8], this can be transformed into a sine-Gordon theory with

$$\mathcal{L}_{\text{eff}} = \left[\frac{1}{2} (\partial \gamma)^2 + C e^{-S_0} \cos \frac{4\pi}{e} \gamma \right], \quad (2.18)$$

as follows. If we calculate $Z = \int (d\phi) e^{-S_{\text{eff}}}$ by expanding in e^{-S_0} we obtain

$$Z = \sum \prod_{\pm} \frac{dx_i^+ dx_i^-}{n_+! n_-!} [C e^{-S_0}]^{n_+ + n_-} \int (d\gamma) \exp \left(- \left[\frac{1}{2} (\partial \gamma)^2 + \frac{4\pi i \gamma \rho}{e} \right] \right), \quad (2.19)$$

where

$$\rho = \sum_{i=1}^{n_+} \delta(x - x_i) - \sum_{i=1}^{n_-} \delta(x - x_i). \quad (2.20)$$

We may now integrate over γ to obtain the Coulomb gas partition function, since

$$\int (d\gamma) \exp \left(- \int \left[\frac{1}{2} (\partial \gamma)^2 + \frac{4\pi i \gamma \rho}{e} \right] \right) = \exp \left(- \frac{1}{2} \left(\frac{4\pi}{e} \right)^2 \int d^3x d^3y \rho(x) \frac{1}{-\partial^2} \rho(y) \right). \quad (2.21)$$

S_0 is the one-instanton action and A the one-instanton one-loop factor. The field γ should be thought of as representing the photon (which is a scalar in $(2+1)$ dimensions) with $\partial_\mu \gamma \simeq B_\mu$. This can be seen by adding an external “magnetic” charge density. Then

$$\mathcal{L}_{\text{eff}} \rightarrow \mathcal{L}_{\text{eff}} + 2 \frac{2\pi i}{e} \int \gamma \rho_{\text{ex}}; \quad (2.22)$$

upon integrating by parts and using $\partial_\mu B_\mu^{\text{ex}} = 4\pi \rho_{\text{ex}}$, we find

$$\mathcal{L}_{\text{eff}} \rightarrow \mathcal{L}_{\text{eff}} + \int \partial_\mu \gamma B_\mu^{\text{ex}}. \quad (2.23)$$

This extra term should be identified with $\int B_\mu B_\mu^{\text{ex}}$. Thus the kinetic term in $\mathcal{L}_{\text{eff}}$ is simply a rewriting of $\frac{1}{2} \int B_\mu B^\mu$. The potential term is generated by instantons. It can't arise in perturbation theory since in perturbation theory $\partial_\mu B_\mu = \partial^2 \gamma = 0$. Its periodicity arises from the quantization of magnetic charge. Adding a topological charge term to the action corresponds to shifting γ by $(e/4\pi)\alpha$.

Now let us include the fermions. We wish to write down an effective action such that expanding $e^{-S_{\text{eff}}}$ in powers of e^{-S_0} will reproduce the instanton gas contributions to any Green functions in an improved dilute-gas approximation where we include the Coulomb interactions. This can be achieved by replacing the topological angle, α , in our previous fermionic effective lagrangian by $(\alpha + 4\pi\gamma/e)$ and adding the kinetic term $\frac{1}{2}(\partial\gamma)^2$. Expanding $e^{-S_{\text{eff}}}$ in powers of e^{-S_0} and integrating over γ puts in the required Coulomb interactions. Integrating over $\psi, \bar{\psi}$ gives the fermion propagators that arise in the multiple scattering approximation to the instanton determinants.

This effective lagrangian,

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial\gamma)^2 + \bar{\psi} i \not{\partial} \psi + \psi^\text{T} \gamma_0 (m e^{i(4\pi\gamma/e + \alpha)}) \psi + \bar{\psi} (m e^{-i(4\pi\gamma/e + \alpha)}) \gamma_0 \bar{\psi}^\text{T}, \quad (2.24)$$

has all the symmetries of the original theory:

$$\begin{aligned} CP: \quad \psi &\rightarrow i\psi, & \bar{\psi} &\rightarrow i\bar{\psi}, \\ \gamma &\rightarrow \gamma + \frac{1}{4}e; \end{aligned} \quad (2.25)$$

$$\begin{aligned} \text{fermion number:} \quad \psi &\rightarrow e^{i\beta} \psi, & \bar{\psi} &\rightarrow e^{-i\beta} \bar{\psi}, \\ \gamma &\rightarrow \gamma - \frac{e\beta}{2\pi}. \end{aligned} \quad (2.26)$$

Furthermore charge conjugation is a symmetry of $\mathcal{L}_{\text{eff}}$:

$$\begin{aligned} C: \quad \frac{4\pi\gamma}{e} + \alpha &\rightarrow -\left(\frac{4\pi\gamma}{e} + \alpha\right), \\ \psi &\leftrightarrow \bar{\psi}. \end{aligned} \quad (2.27)$$

One might ask whether a potential term for γ might not develop from corrections to this improved dilute-gas approximation, thus destroying the fermion number symmetry. It follows from the index theorem that all non-zero topological charge contributions have zero modes and hence corresponds to fermionic terms in $\mathcal{L}_{\text{eff}}$. On the other hand topological charge zero terms should be independent of γ .

Let us now ask whether these symmetries are spontaneously broken. One may analyze $\mathcal{L}_{\text{eff}}$ perturbatively since the coupling constant is (exponentially) small. This requires expanding about some constant value of γ (for example $\gamma = 0$). But this spontaneously breaks fermion number, and CP , giving the fermions a mass. Note that, in the theory without fermions, the photon has a mass squared of order e^{-S_0} . After adding fermions the photon is exactly massless. In subsequent sections, we

will see that in certain theories with *real*, rather than complex fermions, the photon acquires a mass squared that is of order e^{-2S_0} – that is, non-zero, but far smaller than it would be in the absence of fermions.

We can also easily recover the result – explained at the beginning of this section – that the photon is the Goldstone boson associated with the spontaneous breakdown of fermion number conservation. Indeed, in view of the fact [eq. (2.26)] that a fermion number transformation involves a shift in γ , the fermion number current contains a term $J_\mu \sim \partial_\mu \gamma$, showing that γ is a Goldstone boson.

3. Instantons and supersymmetry

We now turn to instanton effects in supersymmetric theories. These theories must have a P or CP symmetry to keep the photon and photino massless to all orders in perturbation theory [see the appendix]. The simplest model has P and C symmetry; it contains an adjoint matter field Φ and a singlet Φ_S , with superpotential

$$W = g\Phi_S(\Phi^2 - \Phi_0^2). \quad (3.1)$$

All fields have masses of order $g\Phi_0$ except the photon multiplet. The photino will be the goldstino of broken supersymmetry if

$$\langle 0 | \left\{ \bar{Q}_\alpha, \frac{\Phi \cdot \lambda_\alpha}{\phi_0} \right\} | 0 \rangle \neq 0. \quad (3.2)$$

We are using a gauge-invariant representation for the photino, $\Phi \cdot \lambda_\alpha / \phi_0$.

Before proceeding we must turn briefly to the topic of functional integration with real fermions. “Real”, in this context, simply means that for every ψ there is *not* an independent integration variable $\bar{\psi}$. Therefore we will let $\bar{\psi}$ stand for $\psi^T \gamma_0$. The fermionic action may be written

$$S_F = \int d^3x \bar{\psi} M \psi, \quad (3.3)$$

where the operator M is *not* hermitian. We may, however, assume that $\gamma_0 M$ is antisymmetric, since the symmetric part makes no contribution to S_F due to the fermion anticommutation. To obtain the integration measure we expand ψ in a complete set of orthonormal functions,

$$\psi(x) = \sum \xi_i \psi^{(i)}(x). \quad (3.4)$$

It is convenient to choose the eigenfunctions of the hermitian operator $M^+ M$:

$$M^+ M \psi^{(i)} = \lambda_i \psi^{(i)}. \quad (3.5)$$

All non-zero eigenfunctions come in pairs: $\tilde{\psi}^{(i)} \equiv [\gamma_0 M \psi^{(i)}]^* / \sqrt{\lambda_i}$ is an independent eigenfunction of $M^+ M$ with eigenvalue λ_i . This follows immediately from the

antisymmetry of $\gamma_0 \mathbf{M}$. Thus S_F diagonalizes:

$$S_F = 2 \sum_{i,j} \tilde{\xi}_i \xi_j \int d^3 x \tilde{\psi}^{(i)T} \gamma_0 \mathbf{M} \psi^{(j)} = 2 \sum_i \sqrt{\lambda_i} \tilde{\xi}_i \xi_i. \quad (3.6)$$

In a CP invariant background field, such as a monopole, $\mathbf{M}$ is odd under $x \rightarrow -x$. Therefore the $\psi^{(i)}$, $\tilde{\psi}^{(i)}$ are even or odd and the members of a pair ξ_i , $\tilde{\xi}_i$ transform oppositely under CP so $d\tilde{\xi}_i d\xi_i \rightarrow d\tilde{\xi}_i d\xi_i$. Thus the CP transformation of the measure is determined by the zero modes. We do not have an index theorem to help us in this case, but we do know of two zero modes from supersymmetry transformations. In terms of a constant spinor η , they are

$$\begin{aligned} \lambda &= \{\bar{Q} \cdot \eta, \lambda\} = \mathbf{B} \eta, \\ \chi &= \{\bar{Q} \cdot \eta, \chi\} = -\mathbf{F} \eta + i \bar{D} \phi \eta, \\ \chi_s &= \{\bar{Q} \cdot \eta, \chi_s\} = -F_s \eta + i \not{D} \phi \eta. \end{aligned} \quad (3.7)$$

The discrete symmetries applied to these zero modes do not produce new zeromodes. Therefore, we believe that there are no other zero modes. [This assumption is consistent with the Callias index theorem in a variation of this theory to which it applies, discussed below.] Under a CP transformation, $\eta \rightarrow +i\eta$ (this follows from the CP transformation of Q). Thus $d^2 \eta \rightarrow -d^2 \eta$ and the integration measure is CP odd.

Let us now turn to calculating

$$\langle 0 | \left\{ \bar{Q}_\alpha, \frac{\Phi \cdot \lambda_\alpha}{\phi_0} \right\} | 0 \rangle = \langle 0 | \frac{\chi \cdot \lambda}{\phi_0} | 0 \rangle. \quad (3.8)$$

Substituting the zero modes we obtain, from the one-instanton sector,

$$\langle 0 | \frac{\chi \cdot \lambda}{\phi_0} | 0 \rangle_I = \frac{A}{i\phi_0} e^{-S_0} \int d^3 x \partial_\mu \Phi \cdot \mathbf{B}_\mu = \frac{2A e^{-S_0} 4\pi}{ie}. \quad (3.9)$$

However, the anti-instanton sector gives a cancelling contribution, since the topological charge $(e/4\pi\phi_0) \int d^3 x \partial_\mu \Phi \cdot \mathbf{B}_\mu$ has the opposite sign. This cancellation is a consequence of C invariance which maps the instanton into the anti-instanton. We can apparently prevent this cancellation by breaking C (while preserving CP). The simplest way to do this is to add a supersymmetric topological charge term

$$\frac{2\alpha e}{8\pi\phi_0} \int d^2 \theta \bar{D}_\alpha \Phi \cdot \mathbf{F}_\alpha = \frac{\alpha e}{4\pi\phi_0} \partial_\mu [\Phi \cdot \mathbf{B}_\mu + \frac{1}{2} \bar{\lambda} \cdot \gamma_\mu \chi] \quad (3.10)$$

to the lagrangian. We then find

$$\langle 0 | \frac{\chi \cdot \lambda}{\phi_0} | 0 \rangle = 2A e^{-S_0} \frac{4\pi}{e} \frac{(e^{i\alpha} - e^{-i\alpha})}{i}. \quad (3.11)$$

Apparently, supersymmetry is broken, the photino has become a Goldstino and the vacuum energy density is

$$E_0 = \frac{1}{2} \left(A e^{-s_0} \frac{8\pi}{e} \sin \alpha \right)^2. \quad (3.12)$$

Let us check this scenario by calculating the instanton contribution to the photino two-point function. We find

$$\langle 0 | \frac{\lambda_\alpha \cdot \Phi(x)}{\phi_0} \frac{\lambda_\beta \cdot \Phi(y) \gamma_0}{\phi_0} | 0 \rangle_I = \frac{A e^{-s_0}}{\phi_0^2} \int d^3 z \int d^2 \eta \lambda_\alpha^{\text{cl}} \cdot \Phi^{\text{cl}}(x-z) \bar{\lambda}_\beta^{\text{cl}} \cdot \Phi^{\text{cl}}(y-z) \gamma_0. \quad (3.13)$$

Inserting the zero modes of eq. (3.7) this becomes

$$\frac{A e^{-s_0}}{\phi_0^2} \int d^3 z \Phi \cdot \mathbf{B}(x-z) \Phi \cdot \mathbf{B}(y-z) e^{i\alpha}.$$

In the multiple scattering dilute-gas approximation we are forced to conclude that a non-local mass term has been generated for the photino field, $\Phi \cdot \lambda_\alpha / \phi_0 \equiv \lambda_\alpha$,

$$m(x-y) = \frac{1}{\phi_0^2} \tilde{\delta} \int d^3 z \Phi \cdot \mathbf{B}(x-z) \Phi \cdot \mathbf{B}(y-z) \tilde{\delta} A e^{-s_0} e^{i\alpha}. \quad (3.14)$$

This can be written as a local mass plus higher derivative terms, the local mass being

$$\begin{aligned} m &= \int d^3 x m(x) = \frac{1}{\phi_0^2} \int d^3 z \tilde{\delta} \Phi \cdot \mathbf{B}(z) \int d^3 x \Phi \cdot \mathbf{B} \tilde{\delta}(x) A e^{-s_0} e^{i\alpha} \\ &= A e^{-s_0} \left(\frac{4\pi}{e} \right)^2 e^{i\alpha}. \end{aligned} \quad (3.15)$$

Adding the anti-instanton contribution (which is the complex conjugate by *CTP*) we get

$$m = A e^{-s_0} \left(\frac{4\pi}{e} \right)^2 2 \cos \alpha. \quad (3.16)$$

Thus our effective low-energy lagrangian is

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \frac{1}{2} \bar{\lambda} i \not{\partial} \lambda + A e^{-s_0} \left(\frac{4\pi}{e} \right)^2 \cos(\alpha) \bar{\lambda} \lambda + \frac{1}{2} \left(A \frac{8\pi}{e} \sin(\alpha) e^{-s_0} \right)^2 \\ &\quad + (\text{higher derivative terms}). \end{aligned} \quad (3.17)$$

We have arrived at a contradiction. If the photino is massive it cannot be a Goldstino, supersymmetry is unbroken and the vacuum energy is zero. Eq. (3.17) only makes sense for $\alpha = 0$ (massive photino, unbroken supersymmetry) or $\alpha = \frac{1}{2}\pi$ (massless photino, broken supersymmetry).

There actually is a second paradox here. The α -dependent term that we have added to the action [eq. (3.10)] is the divergence of a *gauge-invariant* operator. Consequently, all physical quantities should be α independent. This is certainly not obeyed by our effective lagrangian (2.17), according to which the vacuum energy and photino mass depend on α .

The resolution of these paradoxes, as in the last section, lies in the classical instanton interactions. Taking these into account, α becomes a dynamical variable and can therefore “relax” to zero. As explained in the last section, the effective lagrangian corresponding to an improved dilute-gas approximation which includes the coulombic instanton interactions contains a scalar field γ representing the photon. Thus we are led to consider

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial\gamma)^2 + \frac{1}{2}\bar{\lambda}i\not{\partial}\lambda + A e^{-S_0} \left(\frac{4\pi}{e}\right)^2 \cos\left(\frac{4\pi}{e}\gamma + \alpha\right) \bar{\lambda}\lambda + \frac{1}{2}\left(\frac{8\pi}{e}A \sin(\alpha) e^{-S_0}\right)^2. \quad (3.18)$$

This $\mathcal{L}_{\text{eff}}$ still isn't quite sensible. A low-energy effective lagrangian should contain all the symmetries of the original lagrangian, which in this case include supersymmetry. Instanton induced spontaneous supersymmetry breaking, if it occurs at all, should arise as tree level spontaneous symmetry breaking in the effective lagrangian. [In (3+1)-dimensional grand unified theories one might hope that instantons associated with the heavy fields generate a supersymmetric low-energy effective lagrangian with D or F terms which break supersymmetry at the tree level.] If a non-supersymmetric effective lagrangian was obtained we would be forced to conclude that supersymmetry is anomalous rather than spontaneously broken. However, the supersymmetry current is known, from considerations of perturbation theory, to be non-anomalous. The $\mathcal{L}_{\text{eff}}$ of eq. (3.18) can be completed into a supersymmetric lagrangian by replacing α by $(4\pi\gamma/e + \alpha)$ in the vacuum energy term:

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \left[\frac{1}{2}(\partial\gamma)^2 + \frac{1}{2}\bar{\lambda}i\not{\partial}\lambda \right] + A e^{-S_0} \left(\frac{4\pi}{e}\right)^2 \cos\left(\frac{4\pi\gamma}{e} + \alpha\right) \bar{\lambda}\lambda \\ & + \frac{1}{2} \left[\frac{8\pi A}{e} \sin\left(\frac{4\pi\gamma}{e} + \alpha\right) e^{-S_0} \right] + (\text{higher derivative terms}). \end{aligned} \quad (3.19)$$

This is a supersymmetric sine-Gordon theory, the photon and photino forming a scalar multiplet

$$\Gamma = \gamma + \tilde{\theta} \cdot \lambda + \frac{1}{2}\tilde{\theta}\theta F, \quad (3.20)$$

with superpotential

$$W = 2A e^{-S_0} \cos\left(\frac{4\pi\gamma}{e} + \alpha\right). \quad (3.21)$$

If $\alpha = 0$, $\mathcal{L}_{\text{eff}}$ has the C , P (and T) symmetries of the original theory:

$$C: \gamma \rightarrow -\gamma, \quad (3.22)$$

$$P: \gamma \rightarrow -\gamma + \frac{1}{4}e,$$

$$\lambda \rightarrow -i\lambda. \quad (3.23)$$

Adding α has not actually broken C but only shifted the reflection point

$$C: \gamma + \frac{\alpha e}{4\pi} \rightarrow -\left(\gamma + \frac{\alpha e}{4\pi}\right). \quad (3.24)$$

We may now consider the symmetry breaking properties of $\mathcal{L}_{\text{eff}}$. A tree-level analysis should be reliable because of the small coupling constant of $O(e^{-S_0})$. Supersymmetry is unbroken (the vacuum energy is zero) and CP is spontaneously broken, producing a mass for the photon and photino, $m = 2A e^{-S_0} (4\pi/e)^2$. Since the potential for γ of $O(e^{-2S_0})$ is so crucial to the symmetry breaking properties of this theory, it would be nice to understand how it arises from instanton effects. It is clear that the three terms

$$-\frac{8\pi^2}{e^2} A^2 e^{-2S_0} e^{2i(4\pi\gamma/e+\alpha)}, \quad -\frac{8\pi^2}{e^2} A^2 e^{-2S_0} e^{-2i(4\pi\gamma/e+\alpha)}, \quad \frac{16\pi^2}{e^2} A^2 e^{-2S_0}$$

arise respectively from the two-instanton, two-anti-instanton and instanton-anti-instanton sectors. The last term would probably be the hardest to derive since the instanton-anti-instanton sector is not topologically distinct from the perturbative vacuum. The two-instanton term is a correction to the ‘‘improved dilute-gas approximation’’ discussed in the last section. While it seems clear that corrections of $O(e^{-2S_0})$ should exist, we have not been able to show explicitly how this term arises. It is at least comforting to note that such corrections are not ruled out by an index theorem, as was the case in the last section. Note that the photon mass is $O(e^{-2S_0})$ while, in the same theory without fermions, it is $O(e^{-S_0})$.

It is, of course, possible to construct more complicated theories in which the photon multiplet is kept massless by P or CP and C is broken by interactions other than a surface term. Furthermore one could consider higher loop corrections to the semiclassical approximation. Provided that the photon and photino are the only massless particles, the effective lagrangian of eq. (3.19) would still arise where the amplitude, A , and phase, α , would have perturbative expansions. This is the only supersymmetric lagrangian for this multiplet whose Yukawa and F terms are linear in $e^{\pm 4\pi i\gamma/e}$ (a requirement which follows from the fact that the topological charge is ± 1). Actually there is one other term that could be added to W that wouldn't destroy the periodicity of the lagrangian, namely a term linear in γ . Such a term would, in fact, break supersymmetry if its coefficient was larger in absolute value than $(4\pi/e)2A e^{-S_0}$. However, since such a term leads to a term in $F \approx \bar{\chi} \cdot \lambda / \phi_0$ which is γ -independent, it could only arise from topological charge-zero effects.

Furthermore, it cannot arise from ordinary perturbation theory since supersymmetry is unbroken to all orders in perturbation theory. Presumably all non-perturbative zero topological charge effects are exponentially small compared to one-instanton effects ($O(e^{-2S_0})$). Therefore such a linear term, if it arose, would not break supersymmetry. More generally, since one-instanton effects give the photino a mass of $O(e^{-S_0})$, still smaller non-perturbative effects cannot make it into a Goldstino.

4. Massless matter fields

So far we have just considered models in which only the photon multiplet is massless in perturbation theory. We now turn our attention to models with other massless fields. As discussed in the appendix, scalar multiplets can be kept massless by parity, which is a bit like a $(2+1)$ -dimensional analogue of discrete chiral symmetry. One might then hope that the matter field fermion could become the Goldstino through instanton effects.

Consider, for example, the model of the last section, with the pseudoscalar singlet Φ_S replaced by a pair of singlets Φ_1, Φ_2 which interchange under parity with superpotential

$$W = [f(\Phi_1) - f(\Phi_2)](\Phi^2 - \Phi_0^2). \quad (4.1)$$

Renormalizability requires f to be a second-order polynomial. The potential

$$V = \frac{1}{2}[f'(\phi_1)^2 + f'(\phi_2)^2](\Phi^2 - \phi_0^2)^2 + \frac{1}{2}[f(\phi_1) - f(\phi_2)]^2 \Phi^2 \quad (4.2)$$

is zero at

$$\Phi^2 = \phi_0^2, \quad \phi_1 = \phi_2.$$

Since V is zero along the line $\phi_1 = \phi_2$ there is a massless multiplet which we will denote by

$$\Phi = \phi + \bar{\theta} \cdot \chi + \frac{1}{2} \bar{\theta} \theta F. \quad (4.3)$$

W is independent of Φ for $\Phi_1 = \Phi_2 (\equiv \Phi)$. Thus χ is a candidate Goldstino. We must determine whether instanton effects give a non-zero value to

$$\frac{1}{2} \langle 0 | \{ \bar{Q}_\alpha, \chi_\alpha \} | 0 \rangle = \langle 0 | F | 0 \rangle. \quad (4.4)$$

Since $F [=f'(\phi)(\Phi^2 - \phi_0^2)]$ is bosonic, it is not immediately obvious that the instanton contribution to $\langle 0 | F | 0 \rangle$ doesn't vanish because of the two zero modes. However, a non-zero value arises from higher loop effects. [This is possible because the zero modes disappear under infinitesimal variations of the boson fields. They are not related to an index theorem.] The simplest way of seeing this is to introduce supersymmetric collective coordinates. This replaces F by $\{ \bar{Q}_\alpha, [Q_\alpha, F] \} = \partial^2 \phi$,

giving

$$\langle 0|F|0\rangle = A e^{-S_0} \int d^3z \partial^2 \phi_{\text{cl}}(z). \quad (4.5)$$

This is non-zero since ϕ is massless so $\phi_{\text{cl}}(z) \rightarrow 1/|z|$ as $|z| \rightarrow \infty$. As in sect. 3 we have obtained a surface term. This, in fact, always happens since we are performing a triple commutator with Q_α . This might seem to induce supersymmetry breaking; however, we are not entitled to specify the asymptotic behavior or vacuum expectation value of ϕ arbitrarily. We must construct an appropriate effective potential to determine the value of ϕ which is undetermined in perturbation theory. Furthermore an instanton calculation seems to give a non-zero mass to the would-be Goldstino, χ :

$$\langle \chi_\alpha(x) \bar{\chi}_\beta(0) \rangle = A e^{-S_0} \int d^3z \chi_\alpha^{\text{cl}}(x-z) \bar{\chi}_\beta^{\text{cl}}(-z), \quad (4.6)$$

where χ^{cl} represents the zero modes. This corresponds to a non-local mass with the local part being

$$\begin{aligned} m &= A e^{-S_0} \int d^3x \bar{\chi} \chi^{\text{cl}} \int d^3z \chi^{\text{cl}} \bar{\chi} \\ &= A e^{-S_0} \left[\int d^3x \partial^2 \phi \right]^2. \end{aligned} \quad (4.7)$$

We also obtain a non-zero $\langle 0|\{\bar{Q}_\alpha, \lambda_\alpha\}|0\rangle$ and λ mass term as in the last section. All these effects can again be summarized by a low-energy effective lagrangian. It now contains two massless multiplets, the photon multiplet Γ and the matter-field Φ . The superpotential is now

$$W = A(\Phi) e^{-S_0(\Phi)} \cos \Gamma. \quad (4.8)$$

This correctly reproduces the 2 mass terms (which are now interpreted as Yukawa couplings) and F terms. $S_0(\Phi)$ and $A(\Phi)$ are the instanton action and one-loop factors written as functions of the boundary condition on ϕ^{cl} at $|x| \rightarrow \infty$. Note that, to lowest order in the coupling constants

$$\frac{d}{d\phi} (A e^{-S_0}) = -A e^{-S_0} \frac{dS_0}{d\phi}, \quad (4.9)$$

$$\frac{dS_0}{d\phi} = \int d^3x \frac{\delta S}{\delta \phi(x)} \frac{d\phi^{\text{cl}}(x)}{d\phi}. \quad (4.10)$$

Because we are varying ϕ at a solution of the Euler–Lagrange equations this reduces to a surface term S_0 ,

$$\frac{dS_0}{d\phi} = - \int d^3x \partial_\mu \left[\partial_\mu \phi^{\text{cl}}(x) \frac{d\phi^{\text{cl}}(x)}{d\phi} \right] = - \int d^3x \partial^2 \phi^{\text{cl}}(x), \quad (4.11)$$

as required by eqs. (4.5) and (4.6).

Now let us ask if supersymmetry can be spontaneously broken in the low-energy theory with the superpotential of eq. (4.8). In fact, let us be more general and consider a superpotential

$$W = G(\Phi_i) \cos\left(\frac{4\pi}{e}\Gamma\right) + H(\Phi_i) \sin\left(\frac{4\pi}{e}\Gamma\right). \quad (4.12)$$

This is the most general instanton generated superpotential consistent with topological charge ± 1 . The Φ_i are all fields which are undetermined in perturbation theory. The equations for unbroken supersymmetry are

$$\begin{aligned} \frac{\partial W}{\partial \Phi_i} &= \partial_i G \cos\left(\frac{4\pi}{e}\Gamma\right) + \partial_i H \sin\left(\frac{4\pi}{e}\Gamma\right) = 0, \\ \frac{e}{4\pi} \frac{\partial W}{\partial \Gamma} &= -G \sin\left(\frac{4\pi}{e}\Gamma\right) + H \cos\left(\frac{4\pi}{e}\Gamma\right) = 0. \end{aligned} \quad (4.13)$$

There will be a solution

$$\tan\left(\frac{4\pi}{e}\Gamma\right) = \frac{H}{G} = -\frac{\partial_i G}{\partial_i H}, \quad (4.14)$$

provided that a solution of

$$\partial_i (G^2 + H^2) = 0 \quad (4.15)$$

exists. A smooth positive function must be stationary somewhere, although this may occur only at infinity. Thus, the best we may hope is that there is no zero-energy vacuum state except at infinite field strength. This would lead to a cosmological state where the “vacuum” expectation values of the fields (and the constants of nature) are changing with time. Such behavior may occur in some models at the tree level. The difference here is that the “vacuum” energy may initially be exponentially small, and then get smaller still as time goes on. Now let us see if this type of cosmological supersymmetry can occur in the model of eq. (4.1). In this case the function $H(\Phi_i) = 0$ and $G(\Phi) = A(\Phi) e^{-S(\Phi)}$. Thus we must have

$$\frac{dG}{d\phi} = A e^{-S} \int d^3x \partial^2 \phi^{\text{cl}} \neq 0$$

(except possibly at $\phi \rightarrow \infty$). This will presumably be satisfied provided there doesn't exist a monopole solution with ϕ^{cl} constant. Examining the potential [eq. (4.2)], we see that this requires $[f'(\phi)]^2$ to have no stationary point at finite ϕ . This can easily be arranged if we do not require renormalizability [for example $f(\phi) = e^\phi + \phi$]. However, renormalizability requires f to be a second-order polynomial and hence $f'(\phi)^2$ has a minimum at finite ϕ . This difficulty persists even if more fields are added. One might consider models with some additional fields which are undetermined at the tree level but which are not protected by parity. The corresponding

function f would then depend on these fields only through one- (and higher) loop corrections which are not second-order polynomials. However, we suspect that these corrections would tend to give $[f']^2$ a minimum at finite field strength.

Thus our attempts at breaking supersymmetry have met with rather limited success. We start with a model which, in perturbation theory, has a manifold of supersymmetric vacuum state (parameterized by the undetermined scalar fields) and find that instanton corrections generate a non-zero effective potential for the undetermined fields. However, it seems that isolated zero-energy states still exist as well as a zero-energy state at infinite field strength. The type of potential that we obtain is sketched in fig. 3. Of course, if, for some reason, the universe started out in the domain of attraction of the zero-energy state at infinity then exponentially small “cosmological” supersymmetry breaking would result.

5. $N = 2$ supersymmetry

In this section we will discuss models with $N = 2$, or complex, supersymmetry. They provide particularly simple examples of models in which instantons induce a potential for a scalar field whose vacuum expectation value is undetermined in perturbation theory. They also bear a close resemblance to $N = 1$ supersymmetry in $3 + 1$ dimensions. This may be understood as follows.

Consider a minimal $N = 1$ supersymmetric gauge theory in $3 + 1$ dimensions with

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \frac{1}{2} i \bar{\lambda}^a \Gamma_\mu D^\mu \lambda^a \right], \quad (5.1)$$

where Γ_μ are the $3 + 1$ Dirac matrices and λ is a Majorana spinor, $\lambda = C \bar{\lambda}^T$. S is invariant under the supersymmetry transformations

$$\begin{aligned} \delta A_\mu^a &= i \bar{\alpha} \Gamma_\mu \lambda^a, \\ \delta \lambda^a &= \Sigma_{\mu\nu} \alpha F^{\mu\nu a}, \\ \delta \bar{\lambda}^a &= -\bar{\alpha} \Sigma_{\mu\nu} F^{\mu\nu a}, \end{aligned} \quad (5.2)$$

where α is a Majorana spinor and $\Sigma_{\mu\nu} = \frac{1}{4} [\Gamma_\mu, \Gamma_\nu]$.

We can obtain a supersymmetric action in $2 + 1$ dimensions from (5.1) by the process of dimensional reduction. We simply assume all fields to be independent

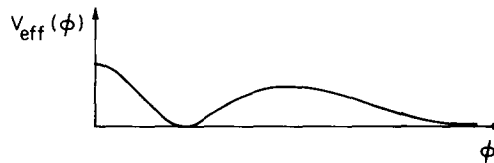


Fig. 3. Typical instanton induced effective potential in $(2+1)$ -dimensional supersymmetric gauge theories with undetermined scalar field expectation values.

of the fourth coordinate and discard all mention of the fourth coordinate from the lagrangian.

The dimensionally reduced version of (5.1) is thus

$$S = \int d^3x \left[-\frac{1}{4} F_{ij}^a F^{ija} + \frac{1}{2} D_i \phi^a D^i \phi^a + i \bar{\chi}^a \gamma_i D^i \chi^a + i f_{abc} \phi^c \bar{\chi}^a \chi^b \right]. \quad (5.3)$$

It is invariant under the complex supersymmetry transformation

$$\begin{aligned} \delta A_i^a &= i \bar{\chi} \gamma_i \alpha + i \bar{\alpha} \gamma_i \chi^a, \\ \delta \phi^a &= i \bar{\chi}^a \alpha - i \bar{\alpha} \chi^a, \\ \delta \chi^a &= -i B^{ia} \gamma_i \alpha - D^i \phi^a \gamma_i \alpha, \end{aligned} \quad (5.4)$$

where α is a two-component complex parameter. Note that the coupling g now has dimensions of $(\text{length})^{-1/2}$.

In the theory described by (5.3) the expectation value of ϕ^a is undetermined at tree level. In general we might expect that perturbative corrections would determine a value for ϕ^a . The fact that this does not happen is due to the presence of non-renormalization theorems in 3+1 dimensions which also hold for the dimensionally reduced theory.

Recall that in 3+1 dimensions superspace has Grassmann coordinates θ and $\bar{\theta}$ which transform as left- and right-handed spinors respectively. Supersymmetric invariants can be formed by integrating a superfield $Q(x, \theta, \bar{\theta})$ over superspace:

$$I_1 = \int d^4x d^2\theta d^2\bar{\theta} Q(x, \theta, \bar{\theta}). \quad (5.5)$$

In addition, if a superfield $R(x, \theta)$ is independent of $\bar{\theta}$ we can form a supersymmetric invariant by integrating only over θ :

$$I_2 = \int d^4x d^2\theta R(x, \theta). \quad (5.6)$$

All mass terms and Yukawa couplings are generated by operators of type I_2 which cannot be written in the form I_1 . Kinetic energy terms for matter and gauge fields can, however, be written in the form I_1 . It has been proved to any finite order of perturbation theory that quantum corrections to the effective potential can only generate operators which can be written in the form I_1 . These non-renormalization theorems are most easily formulated in terms of superspace perturbation theory. In proving them one uses only properties of the θ and θ^* integrations (no particular properties of the momentum integration enter) so these theorems hold also for the dimensionally reduced theory in 2+1 dimensions.

In 2+1 dimensions all spinors have the same transformation properties under Lorentz transformations. There are no left- and right-handed spinors. θ and $\bar{\theta}$ in 3+1 dimensions simply become θ and θ^* in 2+1 dimensions with θ a two-

component complex Grassmann spinor. The structure of superspace perturbation theory is unaltered, however. We simply replace $\bar{\theta}$ by θ^* . There are still two classes of invariants, I_1 and I_2 . Since potential terms for ϕ^a can only be written in the form I_2 , they will not be generated by perturbation theory.

Let us now ask what happens when we include instanton effects. We will find that the non-renormalization theorem is violated at the instanton level.

We will consider the action in (5.3) with a $SU(2)$ gauge group. This model is simply the $V(\Phi) \rightarrow 0$ limit of the model previously considered in sect. 3. The instantons are 't Hooft-Polyakov monopoles in the Prasad-Sommerfeld limit of vanishing Higgs potential. Following the analysis of sect. 3, we would be tempted to conclude that the instanton induced effective action is the same as in eq. (4.8). However, there are several points that need to be considered.

First, since $N = 2$ supersymmetry involves 4 real parameters we would naively expect 4 fermion zero modes given by a supersymmetric transformation on the instanton solution. In euclidean space the supersymmetry transformations are

$$\delta\chi = -(\not{D}\Phi + \not{B})\alpha, \quad \delta\chi^* = -(\not{D}\Phi - \not{B})\alpha. \quad (5.7)$$

For an instanton (anti-instanton) we have $\not{D}\Phi = (\pm)\not{B}$ so that in the presence of an instanton there are two χ zero modes as in the model of sect. 3. This is simply a reflection of the fact that two of the supersymmetry transformations annihilate the instanton as happens in $3+1$ dimensions.

The second point is that for $\langle\Phi\rangle \neq 0$ the massless fields are the photon and a complex fermion, as before, and an additional massless scalar field ϕ . Our low-energy effective lagrangian must take this additional massless field into account. For ϕ constant, $\phi/e > 1$, we know from the explicit Prasad-Sommerfeld solution [12] that the one-instanton action is simply given by $S_0 = 4\pi\phi/e$. To lowest order we can obtain the effective action from that in sect. 4 by writing $S_0 = 4\pi\phi/e$ and adding the kinetic energy term for ϕ :

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu\gamma\partial_\mu\gamma + \partial_\mu\phi\partial_\mu\phi) + \bar{\psi}i\not{\partial}\psi + m e^{-4\pi\phi/e}(\psi^\text{T}\gamma_0\psi e^{i4\pi\gamma/e} + \bar{\psi}\gamma_0\bar{\psi}^\text{T} e^{-i4\pi\gamma/e}). \quad (5.8)$$

If we define $Z = \phi + i\gamma$ we can write

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu Z^*\partial_\mu Z + \bar{\psi}\not{\partial}\psi + m(\psi^\text{T}\gamma_0\psi e^{-4\pi Z^*/e} + \bar{\psi}\gamma_0\bar{\psi}^\text{T} e^{-4\pi Z/e}). \quad (5.9)$$

As before we now argue that $\mathcal{L}_{\text{eff}}$ should be completed into an $N = 2$ supersymmetric action by adding the required potential term

$$V_{\text{eff}} = m^2 e^{-4\pi/\epsilon(Z+Z^*)} = m^2 e^{-8\pi\phi/e}. \quad (5.10)$$

Note that this is consistent with our claim in sect. 3 that no potential term for γ is allowed due to the Callias index theorem. Since V_{eff} involves no fermion fields

and is independent of γ , it must be generated by interactions of net topological charge zero.

There are two independent checks that $\mathcal{L}_{\text{eff}}$ is in fact the correct effective action. First, the fact that the real fields γ and ϕ combined with the correct coefficient to give a complex scalar field was necessary in order that $\mathcal{L}_{\text{eff}}$ be invariant under $N = 2$ supersymmetry. Second, in the Prasad–Sommerfeld limit the Coulomb repulsion between two monopoles is exactly cancelled by the $1/r$ attraction due to the massless scalar field, while the Coulomb attraction between monopole and antimonopole is doubled [13]. But this is exactly what $\mathcal{L}_{\text{eff}}$ gives us. Since ϕ appears with a real coefficient when it is integrated out we obtain a purely attractive Yukawa force of the same magnitude as the Coulomb force induced by γ .

We started with an action for a $(1, \frac{1}{2}, 0)$ $N = 2$ supermultiplet consisting of a vector, a complex fermion, and a real scalar. Due to the fact that there is no such thing as spin for massless particles in $2 + 1$ dimensions, we were able to write the low-energy effective action in terms of a $(\frac{1}{2}, 0)$ $N = 2$ multiplet consisting of a complex scalar and a complex fermion. Since $\mathcal{L}_{\text{eff}}$ involves Yukawa and potential terms we know that it cannot be written as an invariant of the form I_1 . We thus see that the non-renormalization theorem is violated by instanton effects.

Now let us ask whether or not we have induced supersymmetry breaking. Supersymmetry is unbroken only if $V_{\text{eff}}(\phi)$ has a minimum for which $V_{\text{eff}} = 0$. For $\phi/e \leq 1$ our approximation for $\mathcal{L}_{\text{eff}}$ breaks down. The theory is a strongly coupled non-abelian theory. We do not know how to determine whether or not there is a supersymmetric minimum for $\phi/e \leq 1$. For $\phi/e \gg 1$ our approximation is valid and $V_{\text{eff}} \rightarrow 0$ only for $\phi \rightarrow +\infty$. There is no true vacuum state. If such a situation were to arise in $3 + 1$ dimensions we could imagine a cosmological model in which ϕ decreases as the universe ages. If V_{eff} had no supersymmetric minimum for $\phi/e \leq 1$, or if the initial conditions were such that $\phi/e \gg 1$, then supersymmetry would be broken by an amount that depended on the age of the universe and this breaking would be a purely non-perturbative effect.

It is tempting to think that by adding additional fields to this model one could construct a model in which there is a true ground state with broken supersymmetry. Attempts to achieve this run into difficulties similar to those present in $(3 + 1)$ -dimensional models. The fact that there were only two fermion zero modes in the presence of an instanton was crucial to our analysis. As discussed in ref. [3], two zero modes is just the right number in order to have a non-zero coupling of the supercurrent to the Goldstone fermion. In the minimal $N = 2$ model the fact that there were only two zero modes was due to the dual nature of the instanton solution, i.e. $D_i \phi = \pm B_i$. If additional fields with non-zero vacuum expectation values are added this relation will be destroyed and there will be 4 fermion zero modes. For this reason supersymmetry breaking by instantons in these models, if it occurs at all, must be due to a more complicated mechanism than we have presented here.

6. $\text{tr}(-1)^F$

In this section we would like to show that all our above results, obtained by instanton analysis, are consistent with some general constraints on supersymmetry breaking derived in ref. [5].

The general method is to use the invariance of $\text{tr}(-1)^F$ (where F is fermion number) under changes in the parameters of the theory (including the spatial volume). A non-zero value of this quantity indicates that supersymmetry is unbroken. If $\text{tr}(-1)^F$ can be shown to be non-zero in the zero-couplings zero-volume limit then supersymmetry is unbroken in the original theory.

Let us first consider theories with only the photon multiplet massless. Since the unbroken gauge groups is abelian we may gauge away the zero-momentum modes of the gauge field, in a finite volume leaving only the vacuum and the zero-momentum photino as zero-energy states. Thus $\text{tr}(-1)^F = 0$. This tells us nothing. However, it was also argued in ref. [5] that $\text{tr} C(-1)^F$ is an invariant. Since the vacuum is even and the photino is odd under charge conjugation $\text{tr} C(-1)^F = 2$ in C -invariant theories [such as that of eq. (3.1)] and supersymmetry is unbroken. This is just a restatement of the fact that $\langle 0 | \{ \bar{Q}_\alpha, \lambda_\alpha \} | 0 \rangle$ vanishes by charge conjugation. What of the theories that violate C ? It was shown in ref. [5] that C -violating terms can be transformed away by a “conjugation” operation that leaves the number of zero-energy states fixed. [Unlike the 4-dimensional case, a topological term can be changed by conjugation because it is the derivative of a gauge-invariant operator.] This shows that supersymmetry is unbroken in the models of sect. 3.

The models of sect. 4, with undetermined matter fields, cannot be analyzed in this way. A conjugation operation that gives these fields a non-vanishing potential is illegitimate because it changes the behavior of the quantum wave function at large field strength. We may get around this difficulty by imposing twisted boundary conditions on both the gauge field and adjoint matter-field multiplets:

$$\phi \cdot \sigma(x + \hat{e}_i L) = \phi \cdot \sigma_i \sigma \sigma_i(x), \quad i = 1, 2. \quad (6.1)$$

This gets rid of all zero-momentum modes of the boson fields and imposes $\phi = 0$, $A_\mu = 0$ [no non-trivial topological sectors arise in $(2+1)$ dimensions]. We may now calculate $\text{tr}(-1)^F$ in the resulting theory at weak coupling. For the model of eq. (4.1) the potential for the undetermined singlet field ϕ (with ϕ set equal to 0) is

$$V = \frac{1}{2} [f'(\phi)]^2 \cdot \phi_0^4. \quad (6.2)$$

If this potential has a quadratic zero then the only zero-energy state is the vacuum, $\text{tr}(-1)^F = 1$ and supersymmetry is unbroken. This is a weaker result than that of sect. 4 where we concluded that if the potential of eq. (5.2) has *any* minimum supersymmetry is unbroken. However, this analysis does give new information when applied to the $N = 2$ model of sect. 5. With all adjoint fields frozen at zero by the twisted boundary conditions, only one state exists, the zero-energy vacuum

state, so $\text{tr}(-)^F = 1$ and supersymmetry is unbroken. When the volume goes to infinity, this zero-energy vacuum state presumably remains at $\Phi = 0$ and is presumably an $SU(2)$ symmetric strongly interacting state which is inaccessible to the semiclassical methods of sect. 5. The other zero-energy state at $\Phi^2 = \infty$ has apparently been removed by the twisted boundary conditions and finite volume. [It is barely possible that the state at $\Phi^2 = \infty$ instead moves in to $\Phi = 0$ in the zero-volume limit and that it is the only zero-energy state. However this seems unlikely.]

7. Conclusions

We have shown that, while instantons can produce significant effects in $(2 + 1)$ -dimensional theories they cannot break supersymmetry, except possibly in a cosmological sense. This resulted from a constraint on the form of the instanton-induced effects caused by topological charge quantization. Furthermore, these negative results could have been anticipated, more or less, from general $\text{tr}(-)^F$ arguments.

On the positive side, instanton effects do produce a potential for undetermined fields and can violate non-renormalization theorems in the $N = 2$ model to which they apply. Furthermore, terms are generated by instantons which are forbidden by naive zero-mode counting in cases where the zero modes are not topologically stable. Topological charge quantization presumably doesn't impose analogous restrictions in 4 dimensions (or putting it another way, the instanton interactions, which turn the topological charge angle into a dynamical variable in 3 dimensions are short-range and unimportant in 4 dimensions). Furthermore in 4 dimensions (but not 3) fermions in complex representations are kept massless by gauge and Lorentz invariance. Dynamical supersymmetry breaking may still be a viable possibility in such theories.

Appendix

In $2 + 1$ dimensions the Dirac algebra may be realized by two by two matrices, γ^μ , $\mu = 0, 1, 2$, given in a Majorana basis by

$$\gamma^0 = \sigma_2, \quad \gamma^1 = i\sigma_3, \quad \gamma^2 = i\sigma_1. \quad (\text{A.1})$$

They satisfy the commutation relations

$$\begin{aligned} \{\gamma^\mu, \gamma^\nu\} &= 2g^{\mu\nu}, \quad g^{\mu\nu} = (1, -1, -1), \\ [\gamma^\mu, \gamma^\nu] &= -2i\varepsilon^{\mu\nu\lambda}\gamma_\lambda. \end{aligned} \quad (\text{A.2})$$

We work with two-component real (Majorana) fermions χ_α with $\bar{\chi} = \chi^\top \gamma_0$. A massive Majorana spinor is described by the lagrangian

$$\mathcal{L}_M = \frac{1}{2}i\bar{\chi}\gamma^\mu\partial_\mu\chi + \frac{1}{2}m\bar{\chi}\chi. \quad (\text{A.3})$$

The kinetic energy term in $\mathcal{L}_M$ is invariant under the discrete operations of parity (P) and anti-unitary time reversal (T),

$$\begin{aligned} P: (x^0, x^1, x^2) &\rightarrow (x^0, -x^1, x^2), & \chi &\rightarrow \pm i\gamma_1 \chi, \\ T: (x^0, x^1, x^2) &\rightarrow (-x^0, x^1, x^2), & \chi &\rightarrow \pm i\gamma_0 \chi. \end{aligned} \quad (\text{A.4})$$

Note that the mass term is P and T violating since $\bar{\chi}\chi \rightarrow -\bar{\chi}\chi$ under P or T . This may be understood as follows: Since the fermion is massive we can go to its rest frame where P or T would reverse its spin; but since a Majorana fermion in $2+1$ dimensions has only a single spin state this is impossible. A P and T conserving mass term is allowed if we double the number of degrees of freedom by adding another Majorana fermion with spin opposite to that of the first. Indeed

$$\mathcal{L} = \frac{1}{2}i\bar{\chi}_1\gamma^\mu\partial_\mu\chi_1 + \frac{1}{2}i\bar{\chi}_2\gamma^\mu\partial_\mu\chi_2 + \frac{1}{2}m(\bar{\chi}_1\chi_1 - \bar{\chi}_2\chi_2) \quad (\text{A.5})$$

is invariant under

$$P: \chi_1 \rightarrow i\gamma_1\chi_2, \quad \chi_2 \rightarrow i\gamma_1\chi_1, \quad (\text{A.6})$$

$$T: \chi_1 \rightarrow i\gamma_0\chi_2, \quad \chi_2 \rightarrow i\gamma_0\chi_1. \quad (\text{A.7})$$

Gauge fields in $2+1$ dimensions may also have P, T violating gauge-invariant mass terms as discussed in [6]. For an abelian theory such a mass term has the form

$$\mathcal{L}_\mu = \mu B^\mu A_\mu, \quad (\text{A.8})$$

where B^μ is the dual of the field strength, $B_\mu = \frac{1}{2}\epsilon_{\mu\nu\lambda}F^{\nu\lambda}$ with $F^{\nu\lambda} = \partial^\nu A^\lambda - \partial^\lambda A^\nu$. In a non-abelian theory such a mass term has the form

$$\mathcal{L}_\mu = \mu\epsilon^{\mu\nu\lambda}\text{Tr}(F_{\mu\nu}A_\lambda - \frac{2}{3}A_\mu A_\nu A_\lambda). \quad (\text{A.9})$$

We also require the forms of P and T in euclidean space ($x_1, x_2, x_3 = -ix_0$). We use antihermitian γ matrices in euclidean space:

$$\gamma^1 = i\sigma^3, \quad \gamma^2 = i\sigma^1, \quad \gamma^3 = -i\gamma^0 = -i\sigma^2, \quad (\text{A.10})$$

so that $\partial_M \rightarrow \partial_E$. Since the euclidean Lorentz transformation $\psi \rightarrow e^{\gamma^{\mu\theta}_\mu/2}\psi$ is no longer real, fermions in euclidean space may no longer be regarded as real. In euclidean space it is convenient to combine P with an euclidean space rotation by 180° about the x_1 axis so that

$$P_E: x_\mu \rightarrow -x_\mu, \quad \psi \rightarrow \pm i\gamma_1(i\gamma_0\gamma_2)\psi = \pm i\psi. \quad (\text{A.11})$$

“Time reversal” does not reverse $x_3 = -ix_0$ but simply complex conjugates c-numbers and transforms the fermion fields as

$$T_E: \psi \rightarrow \pm \gamma_3\psi. \quad (\text{A.12})$$

We may also define a charge conjugation operation (C) which takes particles into antiparticles. We first consider an abelian gauge group. Since $\bar{\chi}\gamma_\mu\chi \equiv 0$ for a

Majorana spinor we take a *complex* two-component spinor, ψ_α , minimally coupled to an abelian gauge field:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - ieA_\mu)\psi, \quad (\text{A.13})$$

with $\bar{\psi}^* = \psi^*\gamma_0$. Then in the basis (A.1) charge conjugation is given by

$$C: \psi \rightarrow \psi^*, \quad A_\mu \rightarrow -A_\mu. \quad (\text{A.14})$$

Let us consider the case of SU(2) broken down to U(1) by an adjoint Higgs field Φ . If the lagrangian is invariant under the discrete symmetry

$$R: \Phi \rightarrow -\Phi, \quad (\text{A.15})$$

then the resulting U(1) theory will have a charge conjugation symmetry given by the product of R and a SU(2) gauge transformation:

$$C = Rg, \quad g \in \text{SU}(2). \quad (\text{A.16})$$

In perturbation theory it is convenient to choose $\langle\phi_i\rangle = \phi_0\delta_{i3}$ in which case g may be taken to be a rotation by 180° about the 1-axis in isospin space. The action of C is then given by

$$C: A_\mu \rightarrow -A_\mu, \quad W_\mu \rightarrow W_\mu^*, \quad \Phi \rightarrow \Phi, \quad (\text{A.17})$$

where W_μ is the massive charged vector boson and A_μ is the abelian field, $A_\mu = (1/\phi_0)\Phi \cdot \mathbf{A}_\mu$. In instanton calculations it is convenient to require that

$$\Phi(x) \rightarrow \phi_0\hat{x} \quad \text{as } |\mathbf{x}| \rightarrow \infty, \quad (\text{A.18})$$

in which case g may be taken to be a 180° rotation about the $\hat{\theta}$ direction in isospin space ($\hat{\theta}$ is the polar unit vector). We choose the actions of P and T on Φ and A_μ to be consistent with CPT and to give the correct transformation properties for $A_\mu = (1/\phi_0)\Phi \cdot \mathbf{A}_\mu$. If Φ is a scalar rather than a pseudoscalar field then in Minkowski space

$$T: (x^0, x^1, x^2) \rightarrow (-x^0, x^1, x^2), \quad \mathbf{A}_\mu \rightarrow -\mathbf{A}^\mu, \quad \Phi \rightarrow -\Phi, \quad A_\mu \rightarrow -A^\mu, \quad (\text{A.19})$$

while in euclidean space

$$T_E: (x^1, x^2, x^3) \rightarrow (x^1, x^2, x^3), \quad \mathbf{A}_\mu \rightarrow \mathbf{A}_\mu, \quad \Phi \rightarrow -\Phi, \quad A_\mu \rightarrow -A_\mu. \quad (\text{A.20})$$

In 2+1 dimensions the simplest form of supersymmetry involves two real spinor supercharges Q_α , obeying

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\gamma_{\alpha\beta}^\mu P_\mu. \quad (\text{A.21})$$

Superspace is described by three spacetime coordinates and by two real anticommut-

ing variables θ_α . In superspace the Q_α may be represented by

$$Q_\alpha = \frac{\partial}{\partial \bar{\theta}^\alpha} + i(\not{\theta})_\alpha. \quad (\text{A.22})$$

We also introduce supercovariant derivatives

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} - i(\not{\theta})_\alpha. \quad (\text{A.23})$$

A scalar supermultiplet can be written as

$$\Phi = \phi + \bar{\theta}\chi + \frac{1}{2}\bar{\theta}\theta F \quad (\text{A.24})$$

and consists of a real scalar, ϕ , a real spinor χ_α and an auxiliary scalar field F . A vector supermultiplet is described by a real spinor superfield

$$V_\alpha = \psi_\alpha + i(\not{A}\theta)_\alpha + \theta_\alpha D + \frac{1}{2}\bar{\theta}\theta\lambda_\alpha. \quad (\text{A.25})$$

Both D and ψ_α can be gauged away by a supergauge transformation

$$V_\alpha \rightarrow V_\alpha + D_\alpha \Lambda, \quad (\text{A.26})$$

where Λ is a scalar superfield. In Wess-Zumino gauge we then have

$$V_\alpha = i(\not{A}\theta)_\alpha + \frac{1}{2}\bar{\theta}\theta\lambda_\alpha. \quad (\text{A.27})$$

Note that in contradistinction to 3 + 1 dimensions, the vector supermultiplet contains no auxiliary field. The field strength tensor is also described by a spinor superfield

$$F_\alpha = \lambda_\alpha - i(\not{B}\theta)_\alpha - \frac{1}{2}\bar{\theta}\theta(\not{\lambda})_\alpha, \quad (\text{A.28})$$

with $B_\mu = \frac{1}{2}\varepsilon_{\mu\nu\lambda}F^{\nu\lambda}$.

The $\bar{\theta}\theta$ component of any superfield transforms into a total derivative under supersymmetry transformations and may therefore be used to construct a supersymmetric lagrangian. The supersymmetric extension of the SU(2) Higgs model is given by

$$= \int d^2\theta \left[-\frac{1}{2}\mathbf{F}_\alpha \cdot \mathbf{F}_\alpha + \frac{1}{2}(\mathcal{D}_\alpha \Phi) \cdot (\mathcal{D}_\alpha \Phi) + W(\Phi) \right], \quad (\text{A.29})$$

where we have used vector notation for isovectors and $\mathcal{D}_\alpha = D_\alpha - ie\mathbf{V}_\alpha \times$ is the super gauge-covariant derivative. This lagrangian contains the usual gauge-invariant kinetic terms for the vector spinor, and scalar fields, as well as a Higgs potential

$$V = \frac{1}{2} \frac{\partial W}{\partial \phi_i} \frac{\partial W}{\partial \phi_i}, \quad (\text{A.30})$$

and Yukawa couplings

$$\mathcal{L}_y = e\bar{\chi} \cdot (\boldsymbol{\lambda} \times \boldsymbol{\Phi}) + \frac{1}{2}\bar{\chi}_i \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \chi_j. \quad (\text{A.31})$$

The photon mass term (A.8) has a supersymmetric generalization

$$\mathcal{L}_\mu = \mu \bar{F}_\alpha V_\alpha|_{\theta\theta}, \quad (\text{A.32})$$

which gives equal mass to the photon and photino. It can be forbidden by imposing P or CP invariance. The topological charge density can also be written in a supersymmetric form as

$$M = -\frac{i}{8\pi} \bar{D}_\alpha \Phi \cdot F_\alpha|_{\theta\theta} = \frac{1}{4\pi} \partial_\mu [\Phi \cdot B^\mu + \frac{1}{2} \lambda \cdot \gamma^\mu \chi]. \quad (\text{A.33})$$

The Minkowski space P and T transformations can be written in superspace as

$$P: (x^0, x^1, x^2, \theta_\alpha) \rightarrow (x^0, -x^1, x^2, -i(\gamma_1 \theta)_\alpha), \quad \Phi \rightarrow \pm \Phi, \quad V_\alpha \rightarrow i\gamma_1 V_\alpha, \quad (\text{A.34})$$

$$T: (x^0, x^1, x^2, \theta_\alpha) \rightarrow (-x^0, x^1, x^2, i(\gamma_0 \theta)_\alpha), \quad \Phi \rightarrow \pm \Phi, \quad V_\alpha \rightarrow i(\gamma_0 V)_\alpha. \quad (\text{A.35})$$

Since $d^2\theta \rightarrow -d^2\theta$ under the action of P or T , the *superspace* lagrangian must be odd under P or T in a P - or T -invariant theory. In particular the lagrangian in (A.29) is P and T invariant only if $W(\Phi) \equiv 0$ or if additional pseudoscalar fields are added. In the Prasad-Sommerfeld limit $W(\Phi) \equiv 0$, the lagrangian in (A.29) is actually invariant under $N = 2$ supersymmetry. If we introduce a *complex* spinor, $\psi_\alpha = \psi_\alpha + i\lambda_\alpha$, then we have

$$\mathcal{L} = -\frac{1}{2} \mathbf{B}_i \cdot \mathbf{B}^i + \frac{1}{2} D_i \Phi \cdot D^i \Phi + i \bar{\Psi}^* \cdot \gamma^i D_i \Psi + \Psi^* \times (\Psi \times \Phi), \quad (\text{A.36})$$

which is invariant under the $N = 2$ supersymmetry transformations

$$\begin{aligned} \delta_\eta \Phi &= i \bar{\Psi}^* \eta - i \bar{\eta} \Psi, \\ \delta \mathbf{A}_i &= i \bar{\Psi}^* \gamma_i \eta + i \bar{\eta} \gamma_i \Psi, \\ \delta \Psi &= -\gamma^i D_i \Phi \eta - i \gamma^i \mathbf{B}_i \eta, \end{aligned} \quad (\text{A.37})$$

with η a two-component *complex* spinor. The lagrangian may also be derived by dimensional reduction of the lagrangian for a $N = 1$ supersymmetric $SU(2)$ gauge theory in $3 + 1$ dimensions.

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